| **Course:** General algebra | | |
|---|---|---|
| **Course instructors:** Miroslav D. Ćirić, Andreja P. Tepavčević | | |
| **Course type:** Elective | | |
| **Credit points ECTS:** 12 | | |
| **Prerequisites:** No | | |
| **Course objectives:**<br>Acquiring advanced knowledge of basic concepts of universal algebra and basic universal algebraic constructions, ordered sets, lattices, semigroups, groups, rings and modules. | | |
| **Learning outcomes:**<br>Upon completion of the course, the student should master advanced knowledge in the field of universal algebra, ordered sets, networks, semigroups, groups, rings and modules, and be able to apply this knowledge in scientific research in these or other areas. | | |
| **Course description (outline):**<br>Ordered sets, ideals and filters, isotonic functions, residuated functions, closure and opening operators, Galois connections, lattices, sublattices and homomorphisms, distributive and modular lattices, complete lattices, algebraic lattices, algebraic operations, definition and examples of algebras, subalgebras, congruences and quotient algebras, homomorphisms and isomorphisms, basic algebraic constructions, direct and subdirect products, pullback products, products associated with direct products, direct and inverse limits, operators on classes of algebras, varieties of algebras, terms and term algebras, free algebras, equational logic (equational theories), completely invariant congruences, connections with model theory, semigroups, semigroups of transformations and relations, free semigroups, generating sets, monogenic semigroups, groups, homomorphisms of groups, normal subgroups and quotient groups, permutation groups, permutational representation of groups, direct product of groups, cyclic groups, Abelian groups, finitely generated Abelian groups, Sylow theorems and finite groups of small order, free groups, free product groups, group representations, rings, subrings, ring homomorphisms, ring congruences, ideals, quantitative rings, integral domains, unique factorization domains, main ideal domains, Euclidean domains, modules, submodules, homomorphisms of modules, free modules. | | |
| **References:**<br>1. S. Burris, H.P. Sankappanavar, A Course in Universal Algebra, Springer, New York, 1981.<br>2. G. Grätzer, Universal Algebra, Second edition, Springer, New York, 2008.<br>3. J. J. Rotman, An Introduction to the Theory of Groups, Springer, New York, 1994.<br>4. J. J. Rotman, Advanced Modern Algebra, Prentice Hall, 2003.<br>5. S. Crvenković, I. Dolinka, R. Sz. Madarász: Selected topics of general algebra - groups, rings, fields, lattices (in Serbian), Univerzitet u Novom Sadu, 1998. | | |
| Active teaching hours: 5 | Theoretical classes: 5 | Practice classes: |
| **Methods of teaching:**<br>*The lectures use classical teaching methods with the use of modern information and communication technologies and interaction with students. Students' knowledge is tested through homework and defense of seminar papers. The final oral exam checks the comprehensive understanding of the presented material.* | | |
| **Grading structure (100 points)**<br>Activity during the lectures: *10 points*; homework and seminars: *20 points*; oral exam: *70 points*. | | |

**Course:** Theory of ordered sets

**Course instructors:** Andreja P. Tepavčević

**Course type:** Elective

**Credit points ECTS:** 12

**Prerequisites:** No

**Course objectives:**
Introducing students to the most important orderings in mathematics, their properties and role in other mathematical disciplines.

**Learning outcomes:**
Minimal: Understanding the fundamental concepts and properties of ordered sets.
Desirable: Ability to independently and creatively solve more complex problems from ordered sets and a deeper under-standing of all significant features of ordered sets.

**Course description (outline):**
Basic concepts and results: fixed points, closure operators; completion.
Chains and anti-chains. Well-orderings.
Linear orders and linear extensions.
Products of orders and cardinal degree. Lattices.
Complete, algebraic and compact ordered sets.

**References:**


1. B.S.W. Schröder, *Ordered sets*, an Introduction, Birkhäusser, 2003.
2. E. Harzheim, *Ordered Sets*, Springer, 2005.
3. M. Erne, *Algebraic ordered sets and their generalizations*, In: Rosenberg, I., and Sabidussi, G. (eds.), Algebras and Orders. Kluwer, Amsterdam, 1993.


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| Active teaching hours: 5 | Theoretical classes: **5** | Practice classes: |

**Methods of teaching:**
*Theoretical classes with constant interaction with students.*

**Grading structure (100 points)**
Colloquia: *50 points*; oral exam: *50 points*.

**Course:** Semigroup Theory

**Course instructors:** Igor Dolinka, Miroslav Ćirić

**Course type:** elective

**Credit points ECTS:** 10

**Prerequisites:** none

**Course objectives:**

Introduction to fundamental ideas, concepts and results of semigroup theory, as well as applications of semigroups, primarily in other branches of algebra, mathematical logic, and theoretical computer science.

**Learning outcomes:**

Upon completing the course, the student should absorb the basic ideas, concepts and results of semigroup theory. The main goal is to arrive at the operative level of knowledge of the theory, whereupon one can apply it in scientific research within a wide range of mathematical areas.

**Course description (outline):**

*Basic notions of semigroup theory*

Congruences. Rees congruences and ideals. Ideal extensions. Examples of semigroups: free semigroups, transformation semigroups. Presentations of semigroups. Green's relations. The structure of *D*-classes. The Schützenberger group of a *D*-class. Regular *D*-classes.

*Regular semigroups*

Simple and 0-simple semigroups. Principal factors. Completely simple and 0-simple semigroups,the Rees-Suschkevitch theorem. Congruences on completely 0-simple semigroups. Completely regular semigroups (unions of groups). Semilattices of groups. Bands, free bands.

*Introduction to the theory of semigroup decompositions and compositions*

Semilattice decompositions. Band decompositions. Decompositions of semigroups with 0. Subdirect decompositions. Archimedean semigroups and their semilattices. Compositions of semigroups.

*Introduction to the theory of inverse semigroups*

The natural order of inverse semigroups. Congruences of inverse semigroups. Munn's construction. Simple and bi-simple inverse semigroups. *E*-unitary inverse semigroups and McAlister's *P*-theorem. *E*-unitary covers. Factorisability in inverse semigroups. Free inverse monoids.

*Depending on the special interests of students, there is an option for the course to include some current research areas of semigroups theory such as: the theory of ordered semigroups and monoids, combinatorial semigroups theory, the theory of diagram monoids, varieties of semigroups and finite basis problems, pseudovarieties of finite semigroups with applications to automata and formal languages, etc.*

**References:**


1. J.M.Howie, *Fundamentals of Semigroup Theory*, Oxford University Press, New York, 1995.
2. A.H.Clifford, G.B.Preston, *The Algebraic Theory of Semigroups*, American Mathematical Society, Vol. 1, 1961, Vol.2, 1967.
3. M.Petrich, *Introduction to Semigroups*, Merrill Publishing Company, Columbus, Ohio, 1973.
4. J.Rhodes, B Steinberg, *The q-theory of Finite Semigroups*, Springer, New York, 2009.
5. M.Petrich, N.R.Reilly, *Completely Regular Semigroups*, Wiley-Interscience Publication, 1999.
6. S.Bogdanović, M.Ćirić, *Polugrupe*, Prosveta, Niš, 1993.


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| Active teaching hours: 5 | Theoretical classes: 5 | Practice classes: |

**Methods of teaching:**

The lectures use classical teaching methods, aided by contemporary information and communication technology and interaction with students. Students' progress during the course is monitored by homework assignments and by means of writing and defending seminar papers. The goal of the final oral exam is to test the comprehensive understanding of the material of the course.

**Grading structure (100 points)**

Homework and seminars: 30 points. Final oral exam: 70 points.

**Course:** Universal Algebra

**Course instructors:** Petar Marković

**Course type:** elective

**Credit points ECTS:** 10

**Prerequisites:** none, but desirable previous knowledge of basic Universal Algebra

**Course objectives:**

Introduction to the modern theories of Universal Algebra, particularly the Commutator Theory.

**Learning outcomes:**

Upon completing the course, the student should absorb the basic ideas, concepts and results of Commutator Theory. The main goal is to arrive at the operative level of knowledge of the theory, whereupon one can apply it in scientific research within a wide range of mathematical areas.

**Course description (outline):**

Examples of commutators in groups and rings. Congruence modular varieties and Day terms. Shifting Lemma and its applications. Several definitions of a commutator: centralizer, $[\alpha,\beta]$, $[\alpha,\beta]_s$, $M(\alpha,\beta)$. Basic properties of the commutator. Abelian, strongly Abelian, nilpotent and solvable congruences. Commutator in congruence modular varieties: equivalence of various definitions. Residuated lattice of congruences. Generating $[\alpha,\beta]$ in $A^4$. Abelian and affine algebras in congruence modular varieties. Difference term. Permutability. Gumm terms and congruence modularity. Nilpotent algebras, decomposition and congruence regularity. Rings associated with varieties. Structure of algebras in congruence modular varieties.

**References:**


1. 1. R.Freese, R.N.McKenzie, *Commutator Theory for Congruence Modular Varieties*, Cambridge University Press, 1987.
2. 2. R.N.McKenzie, G.F.McNulty, W.F.Taylor, *Algebras, Lattices, Varieties, I*, Wadsworth and Brooks/Cole, Monterey, 1987.
3. 3. S.Burris, H.P.Sankappanavar, *A Course in Universal Algebra*, Springer-Verlag, 1981.
4. D. Hobby, R. McKenzie, *The Structure of Finite Algebras*, Amer. Math. Soc. 1988.


| Active teaching hours: 5 | Theoretical classes: 5 | Practice classes: |
|---|---|---|

**Methods of teaching:**

The lectures use classical teaching methods, aided by contemporary information and communication technology and interaction with students. Students' progress during the course is monitored by homework assignments and by means of writing and defending seminar papers. The goal of the final oral exam is to test the comprehensive understanding of the material of the course.

**Grading structure (100 points)**

Homework and seminars: 30 points. Final oral exam: 70 points.

Начин провере знања могу бити различити : (писмени испити, усмени испт, презентација пројекта, семинари итд......

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**Course: Ordered algebraic structures**

**Course instructors: Jelena M. Ignjatović, Zorana Z. Jančić**

**Course type: Elective**

**Credit points ECTS: 12**

**Prerequisites: No**

**Course objectives:**

*Acquiring knowledge about various ordered algebraic structures and residuated structures, about their basic applications, and about multi-valued logics based on these structures .*

**Learning outcomes:**

*Upon completion of the course, the student should master the basic ideas, concepts and results in the field of ordered algebraic structures, and be able to apply these ideas, concepts and results independently in scientific research.*

**Course description (outline):**

*Ordered semigroups, lattice-ordered semigroups, natural ordering on a semigroup, ordered semirings, natural ordering on a semiring, dioids, quantales, additively idempotent semirings (path algebras), inclines, residuated algebraic structures, residuated semigroups, residuated semirings, residuated semimodules, residuated lattices, BL-algebras, Heyting algebras, MV-algebras, Gödel algebras, triangular norms on a real unit interval, basic fuzzy structures, fuzzy logic, max-plus, min-plus and max-min algebras.*

**References:**


6. T. S. Blyth, Lattices and Ordered Algebraic Structures, Springer, London, 2005.
7. M. Gondran, M. Minoux, Graphs, Dioids and Semirings – New Models and Algorithms, Springer, Berlin, 2008.
8. G. Birkhoff, Lattice Theory, third ed., American Mathematical Society, Providence, RI, 1973..
9. N. Galatos, P. Jipsen, T. Kowalski, H. Ono, Residuated Lattices - An Algebraic Glimpse at Substructural Logics, Elsevier, 2007.
10. R. Belohlavek, V. Vychodil, Fuzzy Equational Logic, Springer, Berlin-Heidelberg, 2005.
11. R. Belohlávek, Fuzzy Relational Systems: Foundations and Principles, Kluwer Academic Publishers, New York, 2002.


| Active teaching hours: 5 | Theoretical classes: 5 | Practice classes: |
|---|---|---|

**Methods of teaching:**

*The lectures use classical teaching methods with the use of modern information and communication technologies and interaction with students. Students' knowledge is tested through homework and defense of seminar papers. The final oral exam checks the comprehensive understanding of the presented material.*

**Grading structure (100 points)**

Activity during the lectures: *10 points*; homework and seminars: *20 points*; oral exam: *70 points*.

**Course: Semiring theory**

**Course instructors: Nada Ž. Damljanović, Aleksandar B. Stamenković**

**Course type: Elective**

**Credit points ECTS: 12**

**Prerequisites: No**

**Course objectives:**

*Introduction to the basic ideas, concepts and results of semiring theory, as well as introduction to the applications of semi-rings.*

**Learning outcomes:**

*Upon completion of the course, the student should master the basic ideas, concepts and results of semiring theory, and be able to apply these ideas, concepts and results independently in scientific research within the same or another scientific field.*

**Course description (outline):**

*Semirings, ordered semirings, complete semirings, star operation, continuous semirings, power series over semirings, rational power series, semimodules, residuated semirings and semimodules, diodes, anti-rings, additively idempotent semirings, inclines, max-plus, min-plus and max-min algebras, matrix calculus over semirings, transitive closures, linear dependence and independence in semimodules, eigenvectors and subeigenvectors, solving systems of linear equations and inequations, solving matrix inequations and equations over diodes, max-plus, min-plus and max-min algebras, applications in optimiza-tion, data analysis and other fields, diodes and nonlinear analysis.*

**References:**


12. J. Golan, Semirings and Their Applications. Kluwer Academic, Dordrecht, 1999.
13. M. Gondran, M. Minoux, Graphs, Dioids and Semirings – New Models and Algorithms, Springer, Berlin, 2008.
14. P. Butkovič, Max-linear Systems: Theory and Algorithms, Springer, London, 2010.
15. B. Heidergott, G.J. Olsder, J. van der Woude, Max Plus at Work: Modeling and Analysis of Synchronized Systems: A Course on Max-Plus Algebra, Princeton University Press, Princeton, 2006.
16. Z. Q. Cao, K. H. Kim, F. W. Roush, Incline Algebra and Applications, John Wiley, New York, 1984.
17. J. Gunawardena, Idempotency, Cambridge University Press, 1998.


| Active teaching hours: 5 | Theoretical classes: 5 | Practice classes: |
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**Methods of teaching:**

*The lectures use classical teaching methods with the use of modern information and communication technologies and interaction with students. Students' knowledge is tested through homework and defense of seminar papers. The final oral exam checks the comprehensive understanding of the presented material.*

**Grading structure (100 points)**

Activity during the lectures: *10 points*; homework and seminars: *20 points*; oral exam: *70 points*.

**Course: Lattice theory**

**Course instructors: Andreja P. Tepavčević**

**Course type: Elective**

**Credit points ECTS: 12**

**Prerequisites: No**

**Course objectives:**

*Introducing students to classical lattice theory, its properties and applications in mathematics. Mastering some special classes of lattices and applications.*

**Learning outcomes:**

*Minimum: Adoption of fundamental concepts and properties of lattices.*

*Desirable: Ability to independently and creatively solve complex problems in lattice theory and its applications in mathe-matics.*

**Course description (outline):**

*Ordered sets and lattices. Lattices as algebras.*

*Complete lattices, algebraic lattices, closure operators. Completion.*

*Modular lattices. Distributive lattices. Complemented and Boolean lattices. Representation theorems.*

*Free lattices. Varieties of lattices. Semimodular and geometric lattices.*

*Continuous lattices. Complete distributivity. Irreducibility. Algebraic lattices. Scott topology.*

**References:**


4. B. Šešelja, Lattice Theory (in Serbian), Departman za matematiku i informatiku, PMF Novi Sad, 2006.
5. B.A. Davey, H.A. Priestley, Introduction to lattices and order. Cambridge Mathematical Textbooks, Cambridge University Press, Cambridge, 1990.
6. G. Gratzer, General Lattice Theory, Second edition, Birkhauser, 2003.
7. G. Birkhoff, Lattice Theory, 3ed, AMS, 1967.
8. R. Freese, J. Jezek, J. B. Nation, Free lattices, Mathematical Surveys and Monographs, 42. American Mathematical Society, Providence, RI, 1995.
9. G. Gierz, K.H. Hofmann, K.Keimel, J. D. Lawson, M. Mislove, D.S. Scott, A compendium of continuous lattices, Springer Verlag 1980.


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| Active teaching hours: 5 | Theoretical classes: 5 | Practice classes: |

**Methods of teaching:**

*Theoretical classes with constant interaction with students.*

**Grading structure (100 points)**

Colloquia: *40 points*; oral exam: *60 points*.

**Course: Fuzzy sets and systems**

**Course instructors: Jelena M. Ignjatović, Ivana Z. Micić**

**Course type: Elective**

**Credit points ECTS: 12**

**Prerequisites: No**

**Course objectives:**

*Introduction to the basic ideas, concepts and results of the theory of fuzzy sets and systems, to the algebraic fundamentals of fuzzy logic, as well as to practical applications of fuzzy sets and methods for solving fuzzy relation equations and inequa-tions.*

**Learning outcomes:**

*After completing the course, the student should master the basic ideas, concepts and results of the theory of fuzzy sets and systems, and be able to independently apply these ideas, concepts and results in scientific research within the same or some other scientific field.*

**Course description (outline):**

*Fuzzy sets: Concept of fuzzy set, set and algebraic operations on fuzzy sets, Principle of extension, fuzzy relations, composi-tion of fuzzy relations, fuzzy orderings, fuzzy equivalences and fuzzy equalities, fuzzy partitions, fuzzy functions, extensio-nality, fuzzy matrices, fuzzy closures. Algebraic basics of the fuzzy logic: residuated lattices, Heyting algebras, BL-algebras, MV-algebras, Gödel algebras, triangular norms on a unit interval, Łukasiewicz, product and Gödel norm.*

*Applications of fuzzy sets: Uncertainty modeling, fuzzy logic and approximate reasoning, fuzzy control, fuzzy data analysis, fuzzy clustering, fuzzy decision making, fuzzy languages and fuzzy automata, fuzzy algebraic structures, fuzzy relational systems, fuzzy graphs, fuzzy topological spaces. Effective procedures for solving szstems of fuzzy relation equations and inequations.*

**References:**


18. R.Belohlavek, Fuzzy Relational Systems: Foundations and Principles, Kluwer Academic Publishers, New York, 2002.
19. R. Belohlavek and V. Vychodil, Fuzzy Equational Logic, Springer, Berlin/Heidelberg, 2005.
20. G. Gerla, Fuzzy Logic: Mathematical Tools for Approximate Reasoning, Kluwer, Dodrecht, 2001.
21. G. J. Klir, B. Yuan, Fuzzy Sets and Fuzzy Logic, Theory and Application, Prentice-Hall, Englewood Cliffs, NJ, 1995.
22. L.-X. Wang, A Course in Fuzzy Systems and Control, Prentice-Hall International, Inc., 1997.


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| Active teaching hours: 5 | Theoretical classes: 5 | Practice classes: |

**Methods of teaching:**

*The lectures use classical teaching methods with the use of modern information and communication technologies and interaction with students. Students' knowledge is tested through homework and defense of seminar papers. The final oral exam checks the comprehensive understanding of the presented material.*

**Grading structure (100 points)**

Activity during the lectures: *10 points*; homework and seminars: *20 points*; oral exam: *70 points*.

**Course:** Group Theory

**Course instructors:** Igor Dolinka, Petar Marković

**Course type:** elective

**Credit points ECTS:** 10

**Prerequisites:** none

**Course objectives:**

Introduction to the fundamental ideas, concepts and results of group theory, including both the core of the classical topics, and the basics of combinatorial group theory as well.

**Learning outcomes:**

Upon completing the course, the student should absorb the basic ideas, concepts and results of group theory. The main goal is to arrive at the operative level of knowledge of the theory, whereupon one can apply it in scientific research within a wide range of mathematical areas.

**Course description (outline):**

Basic properties of groups. Properties of subgroups and normal subgroups, quotient groups and homomorphisms. Isomorphism theorems, the correspondence theorem. Direct and semidirect products of groups. Permutation groups and group actions. Sylow theorems and their applications in the classification of finite groups. Finitely generated abelian groups and the Krull-Schmidt theorem. Normal and composition series. Solvable and nilpotent groups. The extension problem, automorphism groups, wreath products. Simple groups and some classes of linear simple groups.

Free groups and free products. Presentations of groups. Term rewriting systems. Tietze transformations. Subgroups of free products. Generalised free products. Gruško-Neumann's theorem. Geometric methods. Cayley graphs of presentations. Van Kampen diagrams and the Van Kampen lemma. The word problem and the conjugacy problem. Britton's lemma. Dehn's algorithm. Small cancellation theory. One-relator groups and the Magnus-Moldavanskii theory.

**References:**


1. J.J.Rotman, *An Introduction to the Theory of Groups*, , 4th edition, Springer, New York, 1994
2. *M.I.Kargapolov*, Yu.I.*Merzlyakov*, *Fundamentals of the theory of groups*. Springer, New York, 1979.
3. O.Bogopolski, *Introduction to Group Theory*, European Mathematical Society, 2008.
4. R.Lyndon, P.Schupp, *Combinatorial Group Theory*, Springer-Verlag, Berlin, New York, 1977.
5. W.Magnus, A.Karrass, D.Solitar, *Combinatorial Group Theory: Presentations of Groups in Terms of Generators and Relations*, Wiley, New York, 1966.
6. D.L.Johnson, *Presentations of Groups*, Cambridge University Press, 1997.
7. M.Z.Grulović, *Osnovi teorije grupa*, Univerzitet u Novom Sadu, 1997.


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| Active teaching hours: 5 | Theoretical classes: 5 | Practice classes: |

**Methods of teaching:**

The lectures use classical teaching methods, aided by contemporary information and communication technology and interaction with students. Students' progress during the course is monitored by homework assignments and by means of writing and defending seminar papers. The goal of the final oral exam is to test the comprehensive understanding of the material of the course.

**Grading structure (100 points)**

Homework and seminars: 30 points. Final oral exam: 70 points.

**Course: Relational systems**

**Course instructors: Miroslav D. Ćirić, Stefan P. Stanimirović**

**Course type: Elective**

**Credit points ECTS: 12**

**Prerequisites: No**

**Course objectives:**

*Acquiring advanced knowledge of classical Boolean relations, relational systems and relation algebras, their generaliza-tions and basic applications.*

**Learning outcomes:**

*Upon completion of the course, the student should master advanced knowledge of classical Boolean relations, relational systems and relation algebras, their generalizations - fuzzy and weighted relations and relational systems, as well as their applications in the theory of transition systems and automata, network analysis, concept analysis, modal logic and other areas.*

**Course description (outline):**

*Algebra of relations: set operations, conversion, composition, residuals, properties of relations; relations and Boolean matrices; relation algebras: definition and axiomatization, properties of relation algebras. Generalizations of relation alge-bras: residuated lattices and quantales.*

*Generalizations of classical Boolean relations and Boolean relational systems: fuzzy relations and fuzzy relational systems, fuzzy equivalences and fuzzy quasi-orders, quotient fuzzy relational systems, uniform fuzzy relations; weighted relations and weighted relational systems; matrices over semirings, Boolean and fuzzy matrices; semimodules and bisemimodules of relations; solving systems of relational equations and inequations with Boolean, fuzzy and weighted relations.*

*Applications of relational systems: transition systems and automata, quantitative automata - fuzzy automata, weighted auto-mata; network analysis - social networks, traffic, transport, and production networks, other types of networks; concept ana-lysis; relational systems and modal logics, Kripke models; approximation operators and rough sets; relational databases.*

**References:**


23. G. Schmidt, Relational Mathematics (Encyclopedia of Mathematics and its Applications), Cambridge University Press, Cambridge, 2010.
24. S. Givant, Introduction to Relation Algebras, Springer International Publishing, 2017.
25. R. Sz. Madarász, S. Crvenković, Relation Algebras (in Serbian), Matematički Institut SANU, Beograd, 1992.
26. R. Belohlávek, Fuzzy Relational Systems: Foundations and Principles, Kluwer Academic Publishers, New York, 2002.
27. J. Ignjatović, M. Ćirić, Automata and formal languages (in Serbian), Univerzitet u Nišu, Prirodno-matematički fakultet, Niš, 2016.
28. U. Brandes, T. Erlebach (Eds.), Network Analysis: Methodological Foundations, Lecture Notes in Computer Science, vol. 3418, Springer, 2005.


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| Active teaching hours: 5 | Theoretical classes: 5 | Practice classes: |

**Methods of teaching:**

*The lectures use classical teaching methods with the use of modern information and communication technologies and interaction with students. Students' knowledge is tested through homework and defense of seminar papers. The final oral exam checks the comprehensive understanding of the presented material.*

**Grading structure (100 points)**

Activity during the lectures: *10 points*; homework and seminars: *20 points*; oral exam: *70 points*.

**Course:** Lie groups and algebras

**Course instructors:** Vladimir Dragović, Borislav Gajić, Božidar Jovanović, Milena Radnović

**Course type: elective**

**Credit points:** 10 ECTS

**Prerequisites: -**

**Course objectives:**
The course is devoted to the theory of Lie groups and algebras with emphasis on its connections with differential geometry and Hamiltonian dynamics.

**Learning outcomes:**
Students will learnt the fundamental relationship between Lie groups and Lie algebras, the basic of classification of semisimple Lie algebras and compact Lie groups, as well as the structure of symmetric spaces, Lie-Poisson brackets and Euler equations.

**Course description (outline):**

*Theory*

1. Lie groups and algebras, exponential mapping, homomorphisms, subgroups and subalgebras, representations and actions, homogeneous spaces, fundamental group and universal covering of a Lie group,.
2. Killing forms, semisimple, solvable and nilpotent Lie groups and algebras, Lie and Engel theorems, real forms, compact real forms.
3. Root systems and Dynkin diagrams, classification of semisimple Lie Algebras.
4. Compact Lie groups, maximal tori, Weyl group, fundamental group.
5. Symmetric spaces, Cartan decomposition, symmetric spaces of classical groups.
6. Geodesic flows on Lie groups and homogeneous spaces, Li-Poisson bracket, basic examples of integrable systems on Lie algebras

*Practice*

Homework**,** Seminars talks

**References:**


1. S. Helgason, Differential Geometry, Lie Groups, and Symmetric Spaces. AMS, 2001.
2. A. W. Knapp, Lie groups Beyond an Introduction, Birkhauser, 1996.
3. J. F. Adams, Lectures on Lie Groups, University of Chicago Press, 1982.
4. V. V. Gorbatsevich, A. L. Onishchik and E. B. Vinberg, Lie groups and Lie algebras I, Springer, 1993.
5. W. Ziller, Lie Groups. Representation Theory and Symmetric Spaces, University of Pennsylvania, 2010.
6. В. В. Трофимов, А. Т, Фоменко, Алгебра и геометрия интегрируемых гамильтоновых дифференциальных уравнений, Факториал, Москва 1995.


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| **Active teaching hours:** 5 | **Theoretical classes:** 5 | **Practice classes:** |

**Methods of teaching:**
Lectures and practice, with active participation of the students, discussion, seminars, etc.

| **Grading structure (100 points)** | |
|---|---|
| **Pre-exam obligations** | Homework (30 points), Seminar talk (30 points) |
| **Exam** | Oral Exam (40 points) |

**Course: Differential geometry**

**Teacher(s): Mića Stanković, Sanja Konjik**

**Course status:** elective

**ECTS: 12**

**Prerequisites: None**

**Goal:** Mastering the basic concepts of differential geometry

**Outcomes:** The student is able to independently follow the achievements in the field of differential geometry of curves and surfaces.

**Contents**

**Theoretical lectures**

**Curves:** parametric and implicit form. Reparameterization. Arc length, normal and tangent vector field. Osculatory plane. Torsion. Orthonormal reference system. Curvature and torsion in terms of arbitrary parameterization. Frenet's formulas. Fundamental theorem for space curves.

**Surfaces**: Parametric and implicit form, regularity, surface parameterization, surface curves, first basic form, matrix representation of first basic form. Isometry of parameterized surfaces, tangent plane, normal line and Gaussian mapping. Line surfaces, development surfaces. Normal and geodesic curves on surfaces. The second basic form. Asymptotic directions and asymptotic lines. Shape operator. Mean curvature, Gaussian curvature, main curvatures and their relations. Rodriguez equation. Elliptical, parabolic and hyperbolic points of the surface. Euler’s theorem, main curvatures as extreme values of normal curves at a point. Tensor notation. Covariant derivative and Levi-Civita connection. Christoffel symbols. Codazzi equations. Gauss's theorem.

**Recommended bibliography**


1. С. Минчић, Љ. Велимировић: Диференцијална геометрија кривих и површи, ПМФ Ниш, 2007, ИСБН 978-86-83481-34-7
2. Do Carmo, Manfredo P., DIFFERENTIAL GEOMETRY OF CURVES AND SURFACES , Prentice Hall, 1976.
3. 2 (1948) 47-158
4. Alfred Gray: Modern Differential Geometry of Curves and Surfaces with Mathematica, Second Edition, 1997. SCI., NEW YORK 74 NO$3 (1995) 997-1043


| | | |
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| Number of classes per week | Theoretical: 5 | Practical: |

**Methods of teaching:**
Theoretical lectures and independent work of students during practical hours.

Knowledge estimation: **(max 100 points)**
50 points on pre-exam and 50 points on oral exam

**Course: Riemannian manifolds**

**Teacher(s): Ljubica Velimirović, Božidar Jovanović, Milan Zlatanović,**

**Course status:** elective

**ECTS: 12**

**Prerequisites: None**

**Goal:** Mastering the theory of differentiable manifolds and Riemann manifolds.

**Overcomes:** The student is able to successfully master the theory of differentiable manifolds and Riemannian manifolds.

**Contents**

*Theoretical lectures*

DIFFERENTIABLE MANIFOLDS. Definition and examples of differentiable manifolds. Differentiable mappings and product of differentiable manifolds.

TANGENT VECTORS AND TANGENT SPACE AT A POINT OF MANIFOLDS. Tangent vectors, local base. Curves.

VECTOR FIELDS. Definition of vector field, properties, base, local coordinates. Product of vector fields.

TENSORS. Tensor as a polylinear mapping. Tensor as a component system. Tensors in tangent space. Tensor fields.

AFFINE CONNECTION AND COVARIANT DERIVATIVE. Definition of affine connection. Covariant derivative of the vector and scalar field in the direction. Covariant derivative of the covector field. Covariant derivative of the tensor field. Torsion tensor and curvature tensor.

RIEMANNIAN MANIFOLDS. Definition, condition of non-degeneracy. Tangent space. Metric connection and Levi-Civita connection. Curve tensor, Sector curve.

**Recommended bibliography**


5. S. Minčić, Lj. Velimirović, Diferencijalna geometrija mnogostrukosti, PMF u Nišu, 2011.
6. I.I. Karatopraklieva, Diferencijalna geometrija, Univerz. Izdatelstvo, Sofia, 1994.
7. M.P. Do Carmo, Differetnial geometry of curve and surfaces, Institutito de Matematica Pura e Aplicada, Rio de Janeiro, Brazil, 1976.
8. Dragović, V., Milinković, D., Analiza na mnogostrukostima. Primene u geometriji, mehanici, topologiji , Matematički fakultet, Beograd, 2003.


| | | |
|---|---|---|
| Number of classes per week | Theoretical: 4 | Practical: |

**Methods of teaching**

Theoretical lectures and independent work of students during practical hours.

**Knowledge estimation (max 100 points)**

50 points on pre-exam and 50 points on oral exam

| **Course: Semi-Riemannian geometry** | | |
|---|---|---|
| **Teacher(s): Emilija Nešović, Ljubica Velimirović** | | |
| Course status: elective | | |
| ECTS: 12 | | |
| **Prerequisites**: - | | |
| **Goal**<br>Considering smooth manifolds equipped with metric tensor of arbitrary signature. Studding smooth manifolds with non-degenerate metric tensor by applying tensor calculus. | | |
| Goal The students are successfully achieved theoretical knowledge related with smooth manifolds equipped with metric tensor of arbitrary signature and will be able to apply fundamental methods in studying semi-Riemannian manifolds in further research work. | | |
| **Contents**<br><br>**Theoretical lectures:**<br>**Tensors.** The notion of the tensor field. Tensor types. Value of the tensor at the point on a manifold. Tensor components. Contraction. Covariant derivative. Tensor Derivation. Symmetric bilinear forms. Index of symmetric bilinear form. Scalar product on vector space.<br>**Semi-Riemannian manifolds.** The notion of metric tensor on smooth manifold**.** Definition of semi-Riemannian manifold. Causal character of vectors. Isometries. Levi-Civita connection. Geodesic lines. Exponential map. Riemannian curvature tensor of semi-Riemannian manifold. Sectional curvature of semi-Riemannian manifold. Semi-Riemannian surfaces. Type-changing and metric contraction. Frame fields. Some differential operators. Ricci and scalar curvature of semi-Riemannian manifolds. Semi-Riemannian product manifolds. Local isometries of semi-Riemannian manifolds.<br>**Semi-Riemannian submanifolds.** Tangent and normal vector fields. Induced connection on semi-Riemannian submanifold. Geodesics on semi-Riemannian submanifolds. Totally geodesic semi-Riemannian submanifolds. Semi-Riemannian hypersurfaces. Hyperquadrics. The Codazzi equation. Totally umbilic hypersurfaces. Normal connection on semi-Riemannian submanifold. Congruence theorem. Isometric immersion as semi-Riemannian submanifold.<br>**Practical lectures:**<br>Implementation of the theoretically analysed methods. | | |
| **References:**<br>1. B. O'Neill: *Semi-Riemannian Geometry*, Academic Press, New York, 1983.<br>2. S.C. Newman, Semi-Riemannian geometry: The Mathematical Language of General Relativity, John Willey & Sons, 2019.<br>3. B.Y.Chen, Pseudo-Riemannian geometry, delta-invariants and applications, Wold Scientific, Singapore, 2011. | | |
| Number of classes per week | Theoretical: 5 | Practical: |
| Methods of teaching<br>Theoretical lectures and independent work of students during practical hours. | | |
| Knowledge estimation: **(max 100 points)**<br>50 points on pre-exam and 50 points on oral exam | | |

| | | |
|---|---|---|
| **Course:** Introduction to Riemannian Surfaces and Algebraic Curves | | |
| **Course instructors:** Vladimir Dragović, Borislav Gajić, Božidar Jovanović, Milena Radnović | | |
| **Course type: elective** | | |
| **Credit points:** 10 ECTS | | |
| **Prerequisites: -** | | |
| **Course objectives:**<br>Course dedicated to the basics of the theory of Riemann surfaces and algebraic curves with applications in integrable systems. | | |
| **Learning outcomes:**<br>Students will learn the basics of the theory of Riemann surfaces and algebraic curves, Jacobian varieties, elliptic and theta functions and their applications in integrable systems. | | |
| **Course description (outline):**<br>*Theory*<br>1. Riemann surfaces, holomorphic mappings, differential forms.<br>2. Divisors, Poincare-Hopf theorem, Riemann-Hurwitz theorem.<br>3. Line bundles and sheaves on Riemann surfaces.<br>4. Riemann-Roch theorem.<br>5. Algebraic curves, singularities, Bezout's theorem, gender formula<br>6. Normalization. Hypereliptic curves.<br>7. Jacobian variety and Abel's theorem.<br>8. Theta functions and the Jacobi inverse problem.<br>9. Applications of Theta functions in integrable problems of classical mechanics.<br>*Practice*<br>Homework**,** Seminars talks | | |
| **References:**<br>1. P.A. Griffiths, Introduction to Algebraic Curves, AMS, 1989<br>2 P.A. Griffiths, J. Harris Principles of Algebraic Geometry, Wiley, 1994.<br>3. S. Donaldson, Riemannian Surfaces, Oxford University Press, 2011.<br>4. Б. А. Дубровин, “Тэта-функции и нелинейные уравнения”, *УМН*, **36**:2(218) (1981), 11–80.<br>6. В. Драговић, М. Радновић, Понселеови поризми, квадрике и билијари,<br>Завод за уџбенике, Београд, 2012. | | |
| **Active teaching hours: 5** | **Theoretical classes:** 5 | **Practice classes:** |
| **Methods of teaching:**<br>Lectures and practice, with active participation of the students, discussion, seminars, etc. | | |
| **Grading structure (100 points)** | | |
| **Pre-exam obligations** Homework (30 points), Seminar talk (30 points) | | |
| **Exam** Oral Exam (40 points) | | |

**Course:** Symplectic geometry and analytic mechanics

**Course instructors:** Vladimir Dragović, Borislav Gajić, Božidar Jovanović, Milena Radnović

**Course type: elective**

**Credit points:** 10 ECTS

**Prerequisites: -**

**Course objectives:**

The course is devoted to the symplectic and Poisson geometry with emphasis on its connections with theoretical mechanics.

**Learning outcomes:**

Students will learn symplectic geometry through the perspective of theoretical mechanics. They will be able to apply modern geometric techniques in the study of concrete mechanical systems.

**Course description (outline):**

*Theory*

1. Smooth manifolds. Vector fields and differential forms.
2. Principles of mechanics. Lagrange systems. Legendre transformation.
3. Symplectic manifolds. Poisson manifolds. Hamiltonian systems.
4. Completely integrable systems, Liouville-Arnold theorem.
5. Canonical formalism. Hamilton-Jacobi equations. Method of separation of variables.
6. Hamiltonian actions of Lie groups. Symplectic reduction. Poisson reduction.
7. Rigid body dynamics.
8. Elliptic curves and elliptic functions in mechanics.

*Practice*

Homework**,** Seminars talks

**References:**

1. V.I. Arnold: Mathematical methods of classical mechanics, Graduate Texts in Mathematics, 60 Springer 1989.

2. В. Драговић, Д. Милинковић, Анализа на многострукостима, примене у геометрији, механици, топологији, Математички факултет, Београд 2003.

3. P. Liberman, C.-M. Marle, Symplectic geometry and analytical mechanics, Kluwer, 1987.

4. J. Marsden, T. Ratiu, Introduction to Mechanics and Symmetry, Springer-Verlag New York, 1999.

5. A. C. da Silva, Lectures on Symplectic Geometry, LNM 1764, Springer 2008.

6. Болотин С.В., Карапетян А.В., Кугушев Е.И., Трещев Д.В.,Теоретическая механика, 2010.

| **Active teaching hours: 5** | **Theoretical classes:** 5 | **Practice classes:** |
|---|---|---|

**Methods of teaching:**

Lectures and practice, with active participation of the students, discussion, seminars, etc.

**Grading structure (100 points)**

| | |
|---|---|
| **Pre-exam obligations** | Homework (30 points), Seminar talk (30 points) |
| **Exam** | Oral Exam (40 points) |

**Course: Generalized Riemannian spaces**

**Teacher(s): Mića Stanković, Milan Zlatanović**

**Course status:** elective

**ECTS: 12**

**Prerequisites: None**

**Goal**

Mastering the theory of Riemann spaces, generalized Riemannian spaces and spaces of affine connection. Introduction to Kahlerian, generalized Kahlerian and other spaces.

**Outcomes**

The student is able to successfully master the fundamental theorems of the theory of Riemannian, generalized Riemannian, Kahlerian, generalized Kahlerian and other spaces.

**Contents**

**Theoretical lectures**

1. Tensor analysis.
2. Spaces of affine connection.
3. Riemannian spaces.
4. Generalized Riemannian spaces in the sense of Eisenhart.
5. Kahlerian spaces.
6. Geodesic mappings of generalized Riemannian spaces.
7. Almost geodetic mappings of Riemannian and generalized Riemannian spaces.
8. Holomorphically projective mappings of Kahlerianr and generalized Kahlerian spaces.

**Recommended bibliography**


1. M. S. Stanković, Some mappings of the space of non'symmetric affine connection, University of Niš, Faculty of Science, doctoral dissertation, 2001.
2. S. M. Minčić, M. S. Stanković, Lj.S. Velimirović, Generalized Riemannian spaces and spaces of non-symmetric affine connection, Faculty of Science and Mathematics, Niš, 2013.
3. B. Dragović, D. Milinković, Multiple Analysis, Faculty of Mathematics in Belgrade, 2003.
4. N. S. Sinyukov, Geodesic Mappings of Riemannian Spaces, Science, Moscow, 1979.
5. J. Mikeš, Geodesic, F-planar and holomorphic projective mappings of Riemann and affinely connected spaces, Univ. Palacki, Faculty of Natural Sciences, Doctoral dissertation.
6. S.M. Minčić, Generalized Riemann Spaces, Doctoral dissertation, 1976.
7. I. Ivanova-Karatopraklieva, Differential Geometry, Sofia University, 1989.


| | | |
|---|---|---|
| Number of classes per week | Theoretical: 5 | Practical: |

Methods of teaching

Theoretical lectures and independent work of students during practical hours.

**Knowledge estimation (max 100 points)**

50 points on pre-exam and 50 points on oral exam

**Course:** Geodesic mappings

**Teacher(s):** Mića Stanković, Milan Zlatanović

**Course status:** elective

**ECTS:** 12

**Prerequisites:** None

**Goal:** Mastering the theory of geodesic and almost geodetic mappings of Riemann spaces, generalized Riemann spaces and spaces of affine connection. Introduction to mappings of Keller and other spaces.

**Overcomes:** The student is able to successfully master the fundamental theorems of the theory of geodetic, almost geodetic, holomorphic projective conforms and other mappings.

**Contents**

*Theoretical lectures*

1. Introductory terms. Tensor analysis. Spaces of affine connection. Riemannian spaces. Generalized Riemannian spaces. Kahler spaces.
2. Geodesic mappings of Riemannian spaces.
3. Geodesic mappings of special spaces
4. Geodesic mappings of generalized Riemannian spaces.
5. Almost geodetic mappings of Riemannian and generalized Riemannian spaces
6. Holomorphically projective mappings of Kahler and generalized Kahler spaces

**Recommended bibliography**


1. M. S. Stanković, Some mappings of the spaces of nonsymmetric affine connection, University of Niš, Faculty of Sciences and Mathematics, doctoral dissertation, 2001.
2. S. M. Minčić, M. S. Stanković, Lj.S. Velimirović, Generalized Riemannian spaces and spaces of non-symmetric affine connection, Faculty of Science and Mathematics, Niš, 2013.
3. N. S. Sinyukov, Geodesic Mappings of Riemannian Spaces, Science, Moscow, 1979.
4. J. Mikeš, Geodesic, F-planar and holomorphically projective mappings of Riemannian and affinely connected spaces, Univ. Palacki, Faculty of Natural Sciences, Doctoral dissertation.
5. S.M. Minčić, Generalized Riemann Spaces, Doctoral dissertation, 1976.
6. I. Ivanova-Karatopraklieva, Differential Geometry, Sofia University, 1989.


| Number of classes per week | Theoretical: 4 | Practical: |
|---|---|---|

**Methods of teaching**

Theoretical lectures and independent work of students during practical hours.

**Knowledge estimation (max 100 points)**

50 points on pre-exam and 50 points on oral exam

**Course:** Dynamical systems

**Course instructor(s):** Jelena V. Manojlović

**Course type (compulsory/elective):** elective

**ЕЦТС:** 12 ECTS

**Prerequisites:** none

**Course objectives:** Course focuses on nonlinear dynamics with applications. It emphasis on geometric thinking, computational and analytical methods and makes extensive use of demonstration software. It focuses on applications of nonlinear dynamics to different disciplines, e.g., ecology, engineering, neurobiology, and fluid dynamics.

**Learning outcomes:** Student should gain a thorough knowledge of the theory of nonlinear dynamic systems, understand basic concepts related to a geometric and global way of thinking and should be able to use various analytical methods in nonlinear dynamics. In particular, the student should be able to test the stability of nonlinear dynamic systems with the use of software packages for graphic interpretation of the phase portraits.

**Course description (outline):**

- Phase portraits of linear systems in the plane. Topological classification of dynamical systems. Constructing Phase Plane Diagrams.
- Linearization and Hartman's Theorem. Existence and nonexistence of limit cycles in the plane. Poincare-Bendixson Theorem. Poencare map. Stability and Liapunov functions.Center manifold theory. Normal form theory
- Bifurcation of one-dimensional and two-dimensional dynamical systems
- Three-dimensional dynamical systems and chaos: The Rossler system and chaos. The Lorenz equation and attractor. Chua's Circuit.
- Chaos on strange attractors: Lyapunov exponent. Chaotic orbits. Strange attractors.

**References:**


1. L. Perko**,** Differential Equations and Dynamic Systems, Springer, 1991.
2. M.W.Hirisch, S. Smale, R.L. Devaney – Differential equations, Dynamical systems & An Introduction to Chaos, Second Edition, Elsevier Academic Press, 2004.
3. Stephen Lynch, *Dynamical Systems with Applications using Mathematica*, Birkhauser, Boston, 2007.
4. S. H. Strogatz, *Nonlinear Dynamics and Chaos: With Applications to Physics, Biology, Chemistry and Engineering*, Perseus Books Publishing, 1994


| **Active teaching hours: 5** | **Theoretical classes:** 5 | **Practice classes:** |
|---|---|---|

**Methods of teaching:**

Lectures are conducted using conventional teaching methods in interaction with the students. Knowledge of students is tested through homeworks and preparation and defense of seminar papers. Oral exam checks the understanding of the whole course material.

**Grading structure**

| **Pre-exam obligations** | **Points** | **Exam** | **Points** |
|---|---|---|---|
| Colloquia<br>Seminars | **40** | Oral exam | **40** |

| |
|---|
| **Course title:** Regular variation and differential equations |
| **Course instructor(s):** Jelena V. Manojlović |
| **Course type (compulsory/elective):** elective |
| **Credit points:** 12 ECTS |
| **Prerequisites:** none |
| **Course objectives:** Course focuses on theory of regular variation and its applications in the qualitative theory of differential equations. |
| **Learning outcomes:** Student should gain a thorough knowledge of the theory of regular and rapid varying functions and should be able to use this theory to establish the existence of regularly and rapid varying solutions of linear and nonlinear differential equations as well as to establish asymptotic properties of positive solutions. |
| **Course description (outline):**<br>• Properties of regularly and rapid varying functions<br>• The existence of regularly and rapid varying solutions of linear and nonlinear differential equations<br>• Classification and asymptotic properties of solutions of nonlinear differential equations<br>• Asymptotic properties of regularly varying solutions of linear and nonlinear differential equations<br>• Existence and asymptotic properties of regularly and rapid varying solutions of nonlinear differential equations<br>• Existence and asymptotic properties of regularly varying solutions of systems of nonlinear differential equations |
| **References:**<br>5. V. Marić, ***Regular Variation and Differential Equations,*** Springer, 2000.<br>6. N. H. Bingham, C. M. Goldie and J. L. Teugels, ***Regular Variation***, Encyclopedia of Mathematics and its Applications, 27, Cambridge University Press, 19 |

| | | |
|---|---|---|
| **Active teaching hours: 5** | **Theoretical classes:** 5 | **Practice classes:** |
| **Methods of teaching:**<br>Lectures are conducted using conventional teaching methods in interaction with the students. Knowledge of students is tested through homeworks and preparation and defense of seminar papers. Oral exam checks the understanding of the whole course material. | | |

| **Grading structure** | | | |
|---|---|---|---|
| **Pre-exam obligations** | **Points** | **Exam** | **Points** |
| Colloquia | **40** | Oral exam | **40** |
| Seminars | | | |

**Course:** Nonclassical logics

**Course instructors:** Zoran Ognjanović

**Course type:** elective

**Credit points ECTS:** 12

**Prerequisites:**

**Course objectives:**
Introduction to the basic ideas, concepts and results of the theory of nonclassical logics, as well as practical applications in the analysis of formalized problems.

**Learning outcomes:**
At the end of the course, the student will get acquainted with the basic ideas, concepts and results of the theorz of nonclassical logics, and be will able to independently apply these ideas, concepts and results in scientific research within the same or some other scientific field.

**Course description (outline):**
*Theoretical classes*
Modal logics: modal language, Kripke models, reachability relations, model classes, characteristic axioms, completeness theorems, decidability, complexity; linear or branched time temporal logics, epistemic logics; tableau-based proof procedures.
Intuitionist logics: Kripke models, axiomatization, completeness, decidability.
Non-monotonic logics.
Applications in the knowledge and belief representation.
*Practice classes*

**References:**


1. G. E. Hughes, M. J. Cresswell, A Companion to Modal Logic, Methuen, 1984.
2. Joseph Y Halpern, Y. Moses, A guide to completeness and complexity for modal logics of knowledge and belief, *Artificial Intelligence* **54**, 1992, pp. 319-379.
3. Ronald Fagin, Yoram Moses, Moshe Vardi, Joseph Y Halpern, Reasoning About Knowledge, MIT Press, 1995.
4. Melvin Fitting, Intuitionistic logic, model theory and forcing, North-Holland, 1969.
5. Zoran Ognjanović, Nenad Krdžavac, Uvod u teorijsko računarstvo, FON, Beograd, 2004.


| | | |
|---|---|---|
| Active teaching hours: 5 | Theoretical classes: 5 | Practice classes: |

**Methods of teaching:**
Classical teaching methods with video projectors and interaction with students. Students' knowledge is tested through homework and defense of seminar papers. The final oral exam checks the comprehensive understanding of the presented material.

**Grading structure (100 points)**
Pre-exam obligations:
• activity during classes 10 points,
• seminar paper or oral seminar 30 points,
Oral exam 60 points

**Course:** Mathematical Logic

**Course instructors:** Silvia Ghilezan, Zoran Petrić

**Course type:** elective

**Credit points ECTS:** 12

**Prerequisites:**

**Course objectives:**
This is a basic course preceding other specialised courses in Logic. Since the undergraduate curricula concerning this subject are not uniform, a plan is to cover propositional and predicate calculus.

**Learning outcomes:**
After passing the exam, students are familiar with the notions of syntax and semantics of propositional logic and understand the completeness theorem. Concerning the predicate calculus, they are familiar with the notion of operational-relational structure, interpretation, reduction to prenex normal form and the completeness theorem. Moreover, some basic knowledge in the field of Boolean algebra is acquired.

**Course description (outline):**
*Theoretical classes*
1. Formal languages, valuation, tautologies 2. Substitution, replacement of equivalents 3. Formal systems, natural deduction 4. Hilbert system, deduction theorem 5. Completeness of propositional logic 6. Operational-relational structures 7. Language of first order 8. Valuation, free and bound variables 9. Natural deduction for predicate logic 10. Lattices, Boolean algebras 11. Completeness of predicate logic 12. First-order theories
*Practice classes*

**References:**

1. K. Došen, Osnovna logika, manuscript, 2013, http://www.mi.sanu.ac.rs/~kosta/Osnovna%20logika .pdf
2. P. Janičić, Matematička logika u računarstvu, 2008 http://poincare.matf.bg.ac.rs/~janicic//books/mlr.pdf
3. M. Adžić, Beleške iz logike, manuscript, 2021, https://mradzic.github.io/BIL.pdf
4. S.C. Kleene, Mathematical Logic, Dover Publications, New York, 2002
5. E. Mendelson, Introduction to Mathematical Logic, CRC Press, 2010.


| | | |
|---|---|---|
| Active teaching hours: 5 | Theoretical classes: 5 | Practice classes: |

**Methods of teaching:**
Classical teaching methods during lectures are planned. Students evaluation is made through assignments and essays. The final exam is oral and it serves to check the overall comprehension of the presented programme.

**Grading structure (100 points)**
Предиспитне обавезе:
- activities during lectures 10 points,
- essay 30 points,

Final exam 60 points

**Course:** Cryptology I

**Course instructors:** Miodrag Mihaljević

**Course type:** elective

**Credit points ECTS:** 12

**Prerequisites:**

**Course objectives:**
Education on cryptology that provides methods and techniques for information security and privacy.

**Learning outcomes:**
Background on methods and techniques of cryptology for research activates within cryptology, information security and privacy.

**Course description (outline):**

*Theoretical classes*

Introduction and Classical Cryptography, Principles of Modern Cryptography, Provable Security and Real-World Security, Perfectly Secret Encryption, Private-Key (Symmetric) Cryptography, Computational Security, Chosen-Plaintext Attacks and CPA-Security, Chosen-Ciphertext Attacks and CCA-Security, Stream Ciphers, Block Ciphers and Block-Cipher Modes of Operation, Message Authentication Codes, Authenticated Encryption Schemes, Hash Functions and Applications, Practical Constructions of Symmetric-Key Primitives, Theoretical Constructions of Symmetric-Key Primitives, Public-Key (Asymmetric) Cryptography, Number Theory and Cryptographic Hardness Assumptions, Algorithms for Factoring and Computing Discrete Logarithms, Key Management, Public-Key Encryption, Digital Signature Schemes.

*Practice classes*

Exercises from the recommended literature

**References:**

Jonathan Katz, Yehuda Lindell**: *Introduction to Modern Cryptography*,** 3rd Edition**,**
ISBN 9780815354369, Published December, 2020 by Chapman and Hall/CRC, 648 Pages 50 B/W Illustrations

| | | |
|---|---|---|
| Active teaching hours: 5 | Theoretical classes: 5 | Practice classes: |

**Methods of teaching:**
**Consulting, Project Works, Lectures, Homework**

**Grading structure (100 points)**

Pre-Examination activities:

- activity during lectures or consulting: 10 points,
- project work: 30 points,

Oral Examination: 60 points

Начин провере знања могу бити различити : (писмени испити, усмени испт, презентација пројекта, семинари итд......

Oral examination, Project Presentation

*максимална дужна 1 страница А4 формата

**Course:** Model Theory

**Course instructors:** Predrag Tanovic

**Course type:** elective

**Credit points ECTS:** 12

**Prerequisites:**

**Course objectives:** Introduction to the basic ideas and techniques of model theory, as well as applications in other areas of mathematics.

**Learning outcomes:**At the end of the course, the student should master the basic techniques of model theory and be able to apply them in other areas of mathematics, especially in algebra.

**Course description (outline):**

*Theoretical classes*

Definable sets, relations and functions in first order structures. Elementary mappings and extensions. Compactness theorem. Elimination of quantifiers. Types, saturated structures. Omitting types theorem. Homogeneous and universal structures, prime models. Categorical theories.

**References:**


1. David Marker. Model Theory: An Introduction. Graduate texts in mathematics vol.217. Springer 2002.
2. Bruno Poizat. A Course in Model Theory. Springer-Verlag New York 2000.
3. C.C.Chang, H.J.Keisler. Model Theory, 3rd edition. Elsevier Science Publishers. 1990.
4. A.Marcja, C.Toffalori. A guide to Clasiccal and Modern Model Theory. Kluwer Academic Publishers. 2003.


| Active teaching hours: 5 | Theoretical classes: 5 | Practice classes: |
|---|---|---|

**Methods of teaching:**

The lectures use classical teaching methods with the use of video projectors and interaction with students. Students' knowledge is tested through homework and defense of seminar papers. The final oral exam checks the comprehensive understanding of the presented material.

**Grading structure (100 points)**

• activity during the lectures 10 points

• seminar paper or seminar talk held 30 points

• oral exam 60 points

| **Course:** Automated and interactive theorem provers | | |
|---|---|---|
| **Course instructors:** Silvia Ghilezan | | |
| **Course type:** elective | | |
| **Credit points ECTS:** 12 | | |
| **Prerequisites:** Mathematical logic | | |
| **Course objectives:**<br>Introduction to the basic ideas, concepts and results of automated theorem provers and interactive theorem provers (proof assistants), as well as practical applications. | | |
| **Learning outcomes:**<br>At the end of the course, the student will get acquainted with the basic ideas, concepts and results of automated and interactive theorem provers, and will be able to independently apply these ideas, concepts and results in scientific research within the same or some other scientific field. | | |
| **Course description (outline):**<br>Theoretical classes.<br>Automated theorem provers: theoretical foundations, DPLL procedure, resolution method, tableaux method, SAT solvers, SMT solvers.<br>Interactive theorem provers: theoretical foundations, lambda calculus, type theory, dependent types, higher-order logics.<br>Practice classes.<br>Work with automated theorem provers (Prover9, LCF, Z3, Vampire).<br>Work with interactive theorem provers (COQ, Agda, Isabelle). | | |
| **References:**<br>**1.** H.P. Barendregt, Lambda Calculus:Its Syntax and Semantics, North-Holland, 1984<br>**2.** П. Јаничић, Математичка логика у рачунарству, 2008 http://poincare.matf.bg.ac.rs/~janicic//books/mlr.pdf<br>**3.** Ф. Марић, A survey of interactive theorem proving, Zbornik radova, Matematički institut SANU 18(26): 173-223, 2015 http://elib.mi.sanu.ac.rs/files/journals/zr/26/zrn26p173-223.pdf<br>**4.** B. Pierce, Software Foundations, University of Pennsylvania https://softwarefoundations.cis.upenn.edu/current/index.html<br>**5.** The COQ Proof Assistant, https://coq.inria.fr<br>**6.** Isabelle Proof Assistant, https://isabelle.in.tum.de<br>**7.** Agda Proof Assistant, https://wiki.portal.chalmers.se/agda/pmwiki.php | | |
| Active teaching hours: 5 | Theoretical classes: | Practice classes: |
| **Methods of teaching:**<br>Classical teaching methods with video projectors and interaction with students. Students' knowledge is tested through homework and defense of seminar papers. The final oral exam checks the comprehensive understanding of the presented material. | | |
| **Grading structure (100 points)**<br>Pre-exam requrements:<br>• activity during classes 10 points,<br>• seminar paper or oral seminar 30 points.<br>Oral exam 60 points. | | |

**Course:** Formalization of uncertain reasoning

**Course instructors:** Zoran Ognjanović

**Course type:** elective

**Credit points ECTS:** 12

**Prerequisites:**

**Course objectives:**

Introduction to the basic ideas, concepts and results of the theory probability logics, as well as practical applications in uncertain knowledge representation.

**Learning outcomes:**

At the end of the course, the student will get acquainted with the basic ideas, concepts and results абоут пробабилитз логицс, and will be able to independently apply these ideas, concepts and results in scientific research within the same or some other scientific field.

**Course description (outline):**

*Theoretical classes*

Probability logics: models, non-compactness, non-recursive axiomatizations, completeness, decidability, classifications; conditional probability logics (based on Kolmogorov's and Definetti's approaches). Combining probabilistic and other logics: classical, intuitionistic, modal. Applications in the knowledge and beliefs representation, spatial-temporal-probabilistic logic, non-monotonic logic.

*Practice classes*

**References:**


1. Joseph Halpern, Reasoning about Uncertainty. The MIT Press, Cambridge, 2003.
2. Zoran Ognjanović, Miodrag Rašković, Zoran Marković, Probability Logics: Probability-Based Formalization of Uncertain Reasoning, Springer, 2016.
3. Zoran Ognjanović (edt), Probabilistic Extensions of Various Logical Systems. Springer, 2020.
4. Zoran Ognjanović, Nenad Krdžavac, Uvod u teorijsko računarstvo, FON, Beograd, 2004.


| Active teaching hours: 5 | Theoretical classes: 5 | Practice classes: |
|---|---|---|

**Methods of teaching:**

Classical teaching methods with video projectors and interaction with students. Students' knowledge is tested through homework and defense of seminar papers. The final oral exam checks the comprehensive understanding of the presented material.

**Grading structure (100 points)**

Pre-exam obligations:

• activity during classes 10 points,

• seminar paper or oral seminar 30 points,

Oral exam 60 points

| |
|---|
| **Course:** Cryptology II |
| **Course instructors:** Miodrag Mihaljević |
| **Course type:** elective |
| **Credit points ECTS:** 12 |
| **Prerequisites:** exam on Cryptology I |
| **Course objectives:** Additional education within cryptology regarding public key cryptography and quantum cryptography |
| **Learning outcomes:**<br>Background on certain methods and techniques within public key cryptography and quantum cryptography relevant for further research activities |
| **Course description (outline):**<br>*Theoretical classes*:<br>EXPONENTIATION, FACTORING AND DISCRETE LOGARITHMS, Primality testing and integer factorisation using algebraic groups, Basic discrete logarithm algorithms, Factoring and discrete logarithms using pseudorandom walks, Factoring and discrete logarithms in subexponential time, LATTICES, Algorithms for the closest and shortest vector problems, CRYPTOGRAPHY RELATED TO DISCRETE LOGARITHMS, The Diffie–Hellman problem and cryptographic applications, Digital signatures based on discrete logarithms, Public key encryption based on discrete logarithms, CRYPTOGRAPHY RELATED TO INTEGER FACTORISATION, The RSA and Rabin cryptosystems, ADVANCED TOPICS IN ELLIPTIC AND HYPERELLIPTIC CURVES, Isogenies of elliptic curves, Pairings on elliptic curves<br>QUANTUM CRYPTOGRAPHY: Elements of Quantum Information Theory, Quantum Key Distributioн, Quantum Conference Key Agreement, Quantum Key Distribution with Imperfect Devices,<br>Beyond Point-to-Point Quantum Key Distribution, Device-Independent Quantum Cryptography, Quantum Stream Ciphers<br>*Practice classes*: Exercises from the recommended literature |
| **References:**<br>Jonathan Katz, Yehuda Lindell**: *Introduction to Modern Cryptography*,** 3rd Edition,<br>ISBN 9780815354369, Chapman and Hall/CRC, Dec. 2020.<br>Steven D. Galbraith: ***Mathematics of Public Key Cryptography***, Online ISBN: 9781139012843<br>DOI: https://doi.org/10.1017/CBO9781139012843, Cambridge University Press, 2012.<br>Federico Grasselli: ***Quantum Cryptography***, ISBN: 978-3-030-64359-1, Springer, 2021. |

| Active teaching hours: 5 | Theoretical classes: 5 | Practice classes: |
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| **Methods of teaching:**<br>Consulting, Project Works, Lectures, Homework |
| **Grading structure (100 points)**<br>Pre-Examination activities:<br>• activity during lectures or consulting: 10 points,<br>• project work: 30 points,<br>Oral Examination: 60 points |

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|---|---|---|
| **Course:** Blockchain | | |
| **Course instructors:** Miodrag Mihaljević | | |
| **Course type:** elective | | |
| **Credit points ECTS:** 12 | | |
| **Prerequisites:** | | |
| **Course objectives:**<br>Education on blockchain technology and its applications. | | |
| **Learning outcomes:**<br>Background on main methods, techniques, and applications of blockchain technology relevant for research activities. | | |
| **Course description (outline):**<br>*Theoretical classes*:<br>Introduction and Blockchain Paradigms, Technical Basics for Understanding of Blockchain Technology, Overview of Blockchain Technology Concepts, Main Components of a Blockchain System, Architectures of Blockchain Systems, Blockchain Consensus Protocols, Security and Privacy Issues of Blockchain Technologies, Scalability in Blockchain: Challenges and Solutions, Applications of Blockchain Technology, Cryptocurrency Mechanisms for Blockchains: Models, Characteristics, Challenges, and Applications, Blockchain and Internet of Things: An Overview, Cloud-Based Blockchaining for Enhanced Security, Blockchain-Based Security and Privacy for Smart Contracts, Blockchain-Powered Smart Healthcare System, Using Blockchain for Digital Copyrights Management<br><br>*Practice classes*<br>Experimental exercises over selected blockchain platforms | | |
| **References:**<br>Tatiana Gayvoronskaya, Christoph Meinel: ***Blockchain - Hype or Innovation***, Springer, 2021, ISBN 978-3-030-61558-1 ISBN 978-3-030-61559-8 (eBook), https://doi.org/10.1007/978-3-030-61559-8<br>***Handbook of Research on Blockchain Technology***, Edited book, Elsevier, 2020, ISBN: 9780128198162, eBook ISBN: 9780128204153<br>***Blockchain for Information Security and Privacy***, Edited book, Taylor & Francis eBooks, 2021, ISBN 9780367654481 | | |
| Active teaching hours: 5 | Theoretical classes: | Practice classes: |
| **Methods of teaching:**<br>Consulting, Project Works, Lectures | | |
| **Grading structure (100 points)**<br>Pre-Examination activities:<br>• activity during lectures or consulting: 10 points,<br>• project work: 30 points,<br>Oral Examination: 60 points | | |

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| **Course:** Теорија доказа и теорија категорија |
| **Course instructors:** Зоран Петрић |
| **Course type:** elective |
| **Credit points ECTS:** 12 |
| **Prerequisites:** |
| **Course objectives:**<br>Теорија категорија и теорија доказа су повезане у област која носи име Општа теорија доказа. Циљ предмата је да се студент упозна са појмовима кохеренције у категоријама, Генценовим секвентним системима и основним тополошким и алгебарским структурама у којима се могу интерпретирати извођења из различитих формалних система. |
| **Learning outcomes:**<br>Након положеног испита студент влада појмовима категорије, функтора, природне трансформације, лимита и колимита, адјункције, моноидалне категорије, кохеренције и јасна му је техника елиминације сечења. |
| **Course description (outline):**<br>*Theoretical classes*<br>1. Елиминација сечења 2. Категорије, функтори, природне трансформације 3. Универзалне стрелице, лимити и колимити 4. Производи, копроизводи и веза са логиком 5. Адјункција 6. Монаде и моноиди 7. Симплицијална категорија 8. Моноидалне категорије 9. Кохеренција<br>*Practice classes* |
| **References:**<br>1. S. Mac Lane, Categories for the Workong Mathematician, Springer, New York, 1998<br>2. J. Lambek and P.J. Scott, Introduction to Higher Order Categorical Logic, Cambridge University Press, Cambridge, 1986<br>3. K. Dosen and Z. Petric, Proof-Theoretical Coherence, KCL Publications, London, 2004<br>4. K. Dosen and Z. Petric, Proof-Net Categories, Polimetrica, Monza, 2007<br>5. J. Kock, Frobenius Algebras and 2D Topological Quantum Field Theories, Cambridge University Press, Cambridge, 2003 |
| Active teaching hours: 5 \| Theoretical classes: 5 \| Practice classes: |
| **Methods of teaching:**<br>На предавањима се користе класичне методе наставе. Знање студената се тестира преко израде домаћих задатака и одбране семинарских радова. На завршном усменом испиту се проверава свеобухватно разумевање изложеног градива. |
| **Grading structure (100 points)**<br>Предиспитне обавезе:<br>● активност у току предавања 10 поена,<br>● семинарски рад или одржани семинар 30 поена,<br>Усмени испит 60 поена |
| Начин провере знања могу бити различити : (писмени испити, усмени испт, презентација пројекта, семинари итд...... |
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**Course:** Computability theory

**Course instructors:** Silvia Ghilezan, Zoran Ognjanović

**Course type:** elective

**Credit points ECTS:** 12

**Prerequisites:**

**Course objectives:**
Introduction to the basic ideas, concepts and results of the theory of computability and complexity of computation, as well as practical applications in the analysis of formalized problems.

**Learning outcomes:**
At the end of the course, the student will get acquainted with the basic ideas, concepts and results of computability and complexity of computations, and will be able to independently apply these ideas, concepts and results in scientific research within the same or some other scientific field.

**Course description (outline):**
*Theoretical classes*
Basic concepts: coding, recursive functions, Turing machines, equivalence of various formal computational systems, Church thesis.
Computability: Kleenenormal form theorem, decidability, recursively enumerable sets, s-m-n theorem, recursion theorem, relative computability.
Godel's incompleteness theorem: representation of recursive functions and relations in PA, Chinese residual theorem, first and second Godel incompleteness theorems, nondecidability of arithmetic.
Arithmetic hierarchy: the halting problem, jumps, basic definitions and theorems.
Complexity theory: basic definitions, complexity classes, complete problems, probability complexity classes, protocols.
*Practice classes*

**References:**

5. Christos H. Papadimitriou, Harry Lewis, Elements of the theory of computation, Prentice-Hall,1997.
6. Christos H. Papadimitriou, Computational Complexity, Addison Wesley, 1994.
7. Zoran Ognjanović, Nenad Krdžavac, Uvod u teorijsko računarstvo, FON, Beograd, 2004.


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| Active teaching hours: 5 | Theoretical classes: 5 | Practice classes: |

**Methods of teaching:**
Classical teaching methods with video projectors and interaction with students. Students' knowledge is tested through homework and defense of seminar papers. The final oral exam checks the comprehensive understanding of the presented material.

**Grading structure (100 points)**
Pre-exam obligations:
• activity during classes 10 points,
• seminar paper or oral seminar 30 points,
Oral exam 60 points

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|---|---|---|
| **Course:** Analysis on Manifolds | | |
| **Teacher(s):** Sanja Konjik | | |
| **Course status:** elective | | |
| **ECTS:** 12 | | |
| **Prerequisites:** None | | |
| **Goal:** Acquiring knowledge and skills from selected topics of differential geometry on manifolds | | |
| **Outcomes:** Student is able to independently follow the achievements in the field of differential geometry on manifolds and to apply the acquired knowledge and skills to specific problems | | |
| **Contents**<br>*Theoretical lectures*<br>Submanifolds of R^n, immersion, local parametrization, equivalent conditions (local zero set and local graph), smooth maps between submanifolds, chart, differentiable manifolds, maximal atlas, manifold topology, smooth maps between manifolds, topological properties of manifolds, partition of unity, tangent space, tangent map, tangent vector, differentiation, tangent bundle, local vector bundles, vector bundles, sections of vector bundles, vector field, Lie bracket, flow of vector field, integral curve, product of manifolds, submanifolds, embedding, tensors in vector spaces, tensor product, tensor bundle and tensor fields, local representation of tensor fields, alternator, exterior product, exterior algebra, volume element, pullback and push-forward, differential forms, orientation of manifolds, manifolds with boundary, integration on manifolds, Stokes‘ theorem, symplectic vector spaces, symplectic manifolds, Darboux‘s theorem, Hamiltonian vector field, Hamiltonian system, Poisson brackets, Noether‘s theorem, hypersurfaces, Gauss‘ map, Weingarten‘s map, fundamental forms, Riemannian metrics, principal curvatures, Gaussian and mean curvatures, Theorema Egregium, covariant derivative, Cristoffel symbols, intrinsic geometry, parallel transport, geodesics | | |
| **Recommended bibliography**<br>1. Kunzinger, M., Analysis on Manifolds, Lecture notes, University of Vienna, 2022.<br>2. Abraham, R., Marsden, J.E., Foundations of Mechanics, 2nd edition, Addison-Wesley Publishing Company, lnc., USA, 1978.<br>3. Abraham, R., Marsden, J.E., Ratiu, T., Manifolds, Tensor Analysis, and Applications, 2nd edition, Springer-Verlag, New York, 1988.<br>4. Boothby, W.M., An Introduction to Differentiable Manifolds and Riemannian Geometry, Revised 2nd edition, Elsevier Science, USA, 2003.<br>5. Dragović, V., Milinković, D., Analiza na mnogostrukostima, Matematički fakultet, Beograd, 2003. | | |
| Number of classes per week | Theoretical: 5 | Practical: |
| **Methods of teaching:**<br>Theoretical lectures and individual work of students during practical hours | | |
| Knowledge estimation: **(max 100 points)**<br>Mini-project 20 points, oral exam 80 points | | |

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| **Course:** Locally convex spaces |
| **Course instructors:** Stevan Pilipović |
| **Course type:** elective |
| **Credit points ECTS:** 12 |
| **Prerequisites: -** |

**Course objectives:**

Connecting algebraic and topological structures and adopting basic principles of locally convex structures. Understanding of characteristic examples of related structures and application in the study of different classes of operators.

**Learning outcomes:**

To understand basic notions and properties of locally convex spaces: balanced, absorbing and convex sets. Adopting basic principles and peculiarities of different locally convex spaces. The study of tensor products, linear operators, and their connection to kernel functions.

**Course description (outline):**

*Theoretical classes*

Topological vector spaces, local convexity, Frechet spaces. Linear mappings, duality, Radon measures and distributions, tensor products and kernel theorems. Nuclear operators.

**References:**


1. R. Meise, D. Vogt, Introduction to functional analysis, Oxford University Press, Oxford, 1997.
2. H.Schaefer, Topological Vector Spaces, Springer-Verlag, NewYork, 1971.
3. F. Treves, Topological Vector Spaces, Distributions and Kernels, Dover Publications Inc, New York, 2006.


| Active teaching hours: 5 | Theoretical classes: 5 | Practice classes: |
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**Methods of teaching:**

Lectures, discussions and regular consultations

**Grading structure (100 points)**

Solving selected homework: 50 points, oral exam: 50 points

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| **Course:** Time-frequency analysis |
| **Course instructors: :** Nenad Teofanov |
| **Course type:** elective |
| **Credit points ECTS:** 12 |
| **Prerequisites: -** |
| **Course objectives:**<br>Introducing basic notions and techniques of the wavelet transform and time-frequency representations. Understanding the basic principles of signal analysis and synthesis and frame theory. |
| **Learning outcomes:**<br><br>Understanding the basic notions and properties of wavelet bases and frame theory with characteristic examples. Adopting the proofs of basic theorems of time-frequency analysis. Mastering analysis and synthesis techniques in classical cases and in modulation spaces.. |
| **Course description (outline):**<br>*Theoretical classes*<br>Wavelets and multiresolution approximation, Gabor frames and Gabor transform. Modulation spaces. Applications in signal analysis and theory of pseudo-differential operators. |
| **References:**<br>1 K. Gröchenig, K. *Foundations of time-frequency analysis.* Birkhäuser, Boston, 2001.<br>2. I. Daubechies, *Ten Lectures on Wavelets.* SIAM, 1992<br>3. E. Cordero, L. Rodino, *Time-Frequency Analysis of Operators.* de Gruyter, Boston, 2020. |

| Active teaching hours: 5 | Theoretical classes: 5 | Practice classes: |
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| **Methods of teaching:**<br>Lectures, discussions and regular consultations |
| **Grading structure (100 points)**<br>Solving selected homework: 50 points, oral exam: 50 points |

**Course:** Generalized functions

**Course instructors:** Nenad Teofanov, Danijela Rajter-Ćirić

**Course type:** elective

**Credit points ECTS:** 12

**Prerequisites:**

**Course objectives:**

To adopt basic notions from the theory of generalized functions, and observe the ideas leading to its development. To show specific problems which illustrate the importance and applications of the theory.

**Learning outcomes:**

To understand and accept specific features of generalized functions. To master selected practical problems involving the calculus with generalized functions, Fourier transforms and convolutions.

**Course description (outline):**

*Theoretical classes*

Notion and basic properties of test functions and generalized functions (distributions). Fourier transform, convolution, structure theorems and application to partial differential equations. Local and microlocal analysis, propagation of singularities.

**References:**


1. S. Pilipović, B. Stanković, Prostori Distribucija, Srpska Akademija Nauka i Umetnosti, Ogranak u Novom Sadu, Novi Sad, 2000.
2. G. Friedlander, M. Joshi, Introduction to The Theory of distributions, 2nd edition, Cambridge University Press, 1998
3. R. S. Strichartz, A Guide to Distribution Theory and Fourier Transforms, World Scientific, 2003.


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| Active teaching hours: 5 | Theoretical classes: 5 | Practice classes: |

**Methods of teaching:**

Lectures, discussions and regular consultations.

**Grading structure (100 points)**

Solving selected homeworks: 50 points, oral exam: 50 points

**Course:** Integral transforms

**Course instructors:** Diana Dolićanin-Đekić Stevan Pilipović

**Course type:** elective

**Credit points ECTS:** 12

**Prerequisites: -**

**Course objectives:**
Introducing different aspects of the theory if integral transforms. Links to theory of generalized functions and solving differential equations. Adopting basic principles if asymptotic analysis and applications.

**Learning outcomes:**

Connecting different tools of mathematical analysis through applications of integral transforms in time-frequency analysis, differential equations and theory of generalized functions. Insights in basic properties of singular integral operators. Local and global asymptotic analysis of generalized functions through Abelian and Tauberian type theorems. Application to qualitative analysis of solutions to differential equations.

**Course description (outline):**
*Theoretical classes*

Fourier and Laplace transforms and solving equations, convolution, Gaussian integral operators, Hilbert transform and singular integrals. Asymptotic analysis of integral transforms, Abelian and Tauberian type theorems.

**References:**

1. S. Pilipović, B. Stanković, J. Vindas *Asymptotic Behavior of Generalized functions*, World Scientific, Singapore, 2012.
2. A.H. Zemanian, *Generalized Integral Transforms,* John Wiley & Sons, New York, 1968.
3. F.W.King, *Hilbert transforms, Vol 1 and 2, Cambridge University Press, Cambridge,* 2009.
4. Y.A. Neretin, *Lectures on Gaussian Integral Operators and Classical Groups,* EMS, Zurich, 2011.


| Active teaching hours: 5 | Theoretical classes: 5 | Practice classes: |
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**Methods of teaching:**
Lectures, discussions and regular consultations

**Grading structure (100 points)**
Solving selected homework: 50 points, oral exam: 50 points

**Course:** Microlocal Analysis

**Course instructors:** Stevan Pilipović, Nenad Teofanov

**Course type:** elective

**Credit points ECTS:** 12

**Prerequisites: -**

**Course objectives:**

To adopt basic notions and techniques of microlocal analysis. To understand basic principles of propagation of singularities, hypoellipticity and applications to qualitative analysis of partial differential equations and pseudo-differential equations.

**Learning outcomes:**

Understanding the basic principles and techniques of localization and microlocalization. Examples and properties of wave-front sets. Adopting the proofs of basic theorems and understanding of relationship between the classical wave-front sets and its variations. Application to theorems of propagation of singularities.

**Course description (outline):**

*Theoretical classes*

Wave-front sets, generalizations and characterizations, propagation of singularities. Hypoellipticity and application to pseudo-differential operators. Wave front sets and time-frequency analysis.

**References:**


1. G. B. Folland. *Harmonic Analysis in Phase Space*. Princeton Univ. Press, Princeton, NJ, 1989
2. L. Hormander, ¨ *The Analysis of Linear Partial Differential Operators, vol I*, SpringerVerlag, Berlin, 1983.
3. G. Friedlander, M. Joshi, Introduction to The Theory of distributions, 2nd edition, Cambridge University Press, 1998


| Active teaching hours: 5 | Theoretical classes: 5 | Practice classes: |
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**Methods of teaching:**

Lectures, discussions and regular consultations

**Grading structure (100 points)**

Solving selected homework: 50 points, oral exam: 50 points

**Course:** Pseudodifferential and Fourier integral operators

**Course instructors:** Stevan Pilipović

**Course type:** elective

**Credit points ECTS:** 12

**Prerequisites:** -

**Course objectives:**
Introducing basic notions and techniques of theory of pseudo-differential and Fourier integral operators.

**Learning outcomes:**
Students should learn the theory of oscillatory integrals and properties of basic symbol classes, and the problem of quantization from the pseudo-differential calculus point of view. The method of parametrix will provide an understanding of the notion of approximate solutions to some classes of partial differential equations and techniques of qualitative analysis of solutions. It is desirable to master the symbolic calculus and adopt the way to apply symbolic calculus in solving equations.

**Course description (outline):**
*Theoretical classes*
Oscillatory integrals. Basic classes of symbols and the problem of quantization. Fourier integral operators. Algebra of pseudo-differential operators – local and global theories. Pseudo-differential calculus, Weyl and Anti-Wick calculus. Ellipticity and hypoellipticity. Sobolev and Fredholm operator theories.

**References:**

1. F. Treves, Introduction to the theory of pseudodifferential operators and Fourier integral Operators, Plenum Press 1982
2. M.A. Shubin Pseudodifferential operators and spectral theory, Springer-Verlag, Berlin, 1987.
3. Xavier Saint Raymond: Elemetary introduction to the Theory of pseudodifferential operators, CRC Press, 1991
4. F. Nicola, L. Rodino-Global Pseudo-Differential Calculus on Euclidean Spaces, Birknauser, 2010.


| Active teaching hours: 5 | Theoretical classes: 5 | Practice classes: - |
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**Methods of teaching:**
Lectures, discussions and regular consultations

**Grading structure (100 points)**
Solving selected homework: 50 points, oral exam: 50 points

Начин провере знања могу бити различити : (писмени испити, усмени испт, презентација пројекта, семинари итд......

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| **Course:** Numerical integration | | |
| **Course instructors:** Marija Stanić | | |
| **Course type:** elective | | |
| **Credit points ECTS:** 12 | | |
| **Prerequisites: -** | | |
| **Course objectives:**<br>Thorough knowledge and understanding of Approximation theory. Enabling students to solve problems in this area with the use of scientific procedures and methods. Ability to follow modern achievements in the field of Approximation theory and its application. | | |
| **Learning outcomes:**<br>The student has acquired the necessary theoretical knowledge for a systematic understanding of issues related to Approximation theory, its application in other branches of mathematics, technology, and science. The student has mastered the skills and methods of research in this area. | | |
| **Course description (outline):**<br>*Theoretical classes*<br>Basic problems of approximation theory. Uniform mini-max approximations. Mean square approximations. Best $L^1$-approximations. Polynomial and spline approximations. Approximations by rational functions. Extreme problems with algebraic and trigonometric polynomials. Properties of trigonometric and Jacobi polynomial sums.<br><br>*Practice classes*<br>Implementation of the theoretically analysed methods. | | |
| **References:**<br>1. G. Mastroianni, G.V. Milovanovic, *Interpolation Processes – Basic Theory and Applications*, Springer-Verlag, 2008.<br>2. R.A. DeVore, G.G. Lorentz, *Constructive Approximation*, Springer-Verlag, Berlin, 1993.<br>3. G.V. Milovanovic, D.S. Mitrinovic, Th.M. Rassias: *Topics in Polynomials: Extremal Problems, Inequalities, Zeros*, World Scientific Publ. Co., Singapore – New Jersey – London – Hong Kong, 1994. | | |
| Active teaching hours: 5 | Theoretical classes: 5 | Practice classes: |
| **Methods of teaching:**<br>Theoretical lectures and independent work of students during practical hours. | | |
| **Grading structure (100 points)**<br>50 points on pre-exam and 50 points on oral exam | | |

**Course title:** LINEAR PROGRAMMING AND OPTIMIZATION

**Teacher or teachers:** Stanimirović P. Ivan

**course status:** elective

**Credit points ECTS:** 12

**Prerequisites:**

**The aim of the course**

Mastering the knowledge of the use of quantitative methods and optimization algorithms in decision-making in the management process.

**Outcome of the course**

By taking the exam, the student is introduced to the basic concepts and methods of mathematical statistics, principles and methods of optimization that are used to solve management problems; understand that the solutions obtained by mathematical modeling are less burdened than the subjective attitudes of decision makers.

**Course content**

Theoretical classes:

Basic concepts of probability theory and statistics. The decision tree. Sampling, interval estimates and confidence intervals for large and small samples. Determining the confidence interval for the difference, ie the ratio of the considered statistical parameters (arithmetic mean, proportion and variance of the population) of the two populations in the case of small and large samples. Testing of parametric and nonparametric hypotheses. Regression and correlation analysis, application in trend analysis. Systems theory and systems analysis. Introduction to optimization. Mathematical modeling: identification and approximation. Examples of mathematical modeling (mathematical model of the accumulation core, etc.).

One-criteria optimization: definition of the optimization task and optimal solution. An overview of optimization methods. Optimization of stochastic systems. Example of accumulation sizing.

Linear programming (geometric method, simplex method, dual linear programming problem, sensitivity analysis of the obtained solution). Transport problem (closed and open transport task). Scheduling task. Nonlinear programming (unconditional optimization and reduction method). Network planning methods (CPM, PERT and PERT-COST methods).

Multicriteria optimization: problem setting. Methods for determining non-inferior solutions (method of weighting coefficients, method of constraints in the space of criterion functions, multicriteria simplex method). Targeted programming. Interactive methods (STEM and SEMPOPS methods). Stochastic methods (PROTRADE method). Inventory management (basic concepts in inventory management, ABC analysis, deterministic inventory models).

Practical teaching:

Creating assignments that belong to the topic covered in lectures. Illustrative data predominate in the assignments and the goal is for the student to master the processed methods. Practical training of students to use computers in management tasks using available software for all areas studied in this course: software for statistical analysis of problems (Excel, and other software available on the Internet), software for linear programming.

**Recommended literature**


1. I. Stanimirović, Advances in Optimization and Linear Programming, 2021, Apple Academic Press Incorporated, Taylor & Francis.
2. Simonović, V., Tadić, D., Milanović, D., Quantitative methods, ICIM plus, Kruševac, 2005.
3. Jovanović, T., Quantitative methods, Faculty of Mechanical Engineering, Belgrade, 1996.
4. Jovanović, T., et al., Zbirka zadataka iz kvantitativnih metoda, Mašinski fakultet, Beograd, 1996.
5. Tadić, D., Theory of assembly phases-application in solving management problems, Faculty of Mechanical Engineering, University of Kragujevac, Kragujevac, 2006.


Knowledge assessment (maximum number of points 100)

Pre-examination obligations: homework (10 points), seminar paper (30 points), oral exam: 60 points

**Course:** Numerical optimization

**Course instructors:** Nataša Krejić

**Course type:** elective

**Credit points ECTS:** 12

**Prerequisites: -**

**Course objectives:**
Acquiring knowledge on classical methods for unconstrained and constrained optimization problems.

**Learning outcomes:**
The students will be able to proceed with research work on numerical optimization topics and to apply the methdology on relevant problems from other disciplines.

**Course description (outline):**
*Theoretical classes*
Unconstrained problems. Оптимизациони проблеми без ограничења. Necessary and sufficient optimality conditions. Line search methods. Trust region methods. Newton type methods. Least squares methods. Constrained optimization problems. Optimality conditions and theoretical fudnations of algorithms. Small and middle size problems. Large scale problems. Penalty methods. Lagrange multipliers methods. Sequential Quadratic Programming.

*Practice classes*
Implementation of the theoretically analysed methods.

**References:**

1. Nocedal, J. Wright, S.J., Numerical optimization, Springer, 2006.
2. Bertsekas, D.P. Convex Optimization Methods, Athena Scientific, 2015.
3. Birgin, E.G., Martinez, J.M. Practical Augmented Lagranigian Methods for Constrained Optimization, SIAM 2014.


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| Active teaching hours: 5 | Theoretical classes: 5 | Practice classes: |

**Methods of teaching:**
Theoretical lectures and independent work of students during practical hours.

**Grading structure (100 points)**
50 points on pre-exam and 50 points on oral exam

**Course:** Numerical Linear Algebra

**Course instructors:** Marko Petković

**Course type:** elective

**Credit points ECTS:** 12

**Prerequisites: -**

**Course objectives:**

Introduction to the construction and implementation of main numerical methods in linear algebra, and their applications to the practical problems in natural, technical and social sciences.

**Learning outcomes:**

Students should be familiar with the basic and advanced numerical methods in linear algebra. Students should also be able to efficiently implement these methods on solving the concrete practical problems in natural and technical sciences, financial mathematics, etc.

**Course description (outline):**

*Theoretical classes*

- Basic matrix analysis: matrix multiplications, BLAS routines, fast matrix multiplication, parallel algorithms, matrix and vector norms, SVD, stability analysis.
- Linear systems: factorizations (LU, Cholesky, etc.), gaussian elimination, parallel algorithms, structured systems (tridiagonal, banded, Vandermonde, etc.)
- Orthogonalization and least squares: Householder and Givens transformations, QR factorization, regularization, least squares problems, updating matrix factorizations.
- Symmetric and non-symmetric eigenvalue problems: power iterations, QR algorithm, tridiagonal problems, computing SVD, Jacobi methods, sparse problems, Krylov subspace methods, Lanczos method.
- Large sparse linear systems: iterative methods, conjugate gradient, other CG-based methods, preconditioning

*Practice classes*

Implementation of the theoretically analysed methods.

**References:**

[1] G.H. Golub, C.F. Van Loan, Matrix Computations, 4th ed., The Johns Hopkins University Press, Baltimore, 2013.

[2] J. Kiusalaas, Numerical methods in engineering with Python 3, Cambridge University Press, 2013.

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| Active teaching hours: 5 | Theoretical classes: 5 | Practice classes: |

**Methods of teaching:**

Theoretical lectures and independent work of students during practical hours.

**Grading structure (100 points)**

50 points on pre-exam and 50 points on oral exam

**Course:** Numerical integration

**Course instructors:** Marija Stanić

**Course type:** elective

**Credit points ECTS:** 12

**Prerequisites: -**

**Course objectives:**
Thorough knowledge and understanding of quadrature and cubature processes. Enabling students to solve problems in this area with the use of scientific procedures and methods. Ability to follow modern achievements in the field of numerical integration and its application.

**Learning outcomes:**
The student has acquired the necessary theoretical knowledge for a systematic understanding of issues related to the theory of quadrature and cubature formulas, its application in other branches of mathematics, technology, and science. The student has mastered the skills and methods of research in this area.

**Course description (outline):**
*Theoretical classes*
Quadrature formulas of interpolation type. Methods for estimating the remainder. Romberg integration. Gaussian quadrature formulas. Modifications of Gaussian formulas. Radau and Lobatto type formulas. Cronrod's scheme. The existence of formulas. Gauss-Turán quadratures and generalizations. Convergence of quadrature processes. Quadrature formulas with quasi degree of accuracy. Quadrature formulas with maximal trigonometric degree of exactness. Numerical integration of fast oscillatory functions. Interpolation cubature formulas. Construction of formulas based on symmetry. An overview of cubature formulas for some special areas and certain weight functions. Optimal sets of quadrature formulas.

*Practice classes*
Implementation of the theoretically analysed methods.

**References:**

1. P.J. Davis, P. Rabinowitz, *Methods of Numerical Integration,* Academic Press, New York, San Francisco, 1975.
2. H. Engels, *Numerical Quadrature and Qubature*, Academic Press, London, 1980.
3. G. Mastroianni, G.V. Milovanovic, *Interpolation Processes – Basic Theory and Applications*, Springer-Verlag, 2008.
4. W. Gautschi, *Orthogonal Polynomials: Computation and Approximation,* Oxford University Press, Oxford, 2004
5. A. Ghizzetti, A. Ossicini, *Quadrature Formulae*, Akademie - Verlag, Berlin, 1970.


| | | |
|---|---|---|
| Active teaching hours: 5 | Theoretical classes: 5 | Practice classes: |

**Methods of teaching:**
Theoretical lectures and independent work of students during practical hours.

**Grading structure (100 points)**
50 points on pre-exam and 50 points on oral exam

| |
|---|
| **Course:** Multiple Criteria Optimization |
| **Course instructors:** Bogdana Stanojević |
| **Course type:** elective |
| **Credit points ECTS:** 12 |
| **Prerequisites:** Operations Research |
| **Course objectives:**<br>Formal and practical understanding of decision making process in situations where multiple criteria for decision evaluation exist. Three basic cases are considered: when it is needed to chose one decision among a set of predefined decisions, when the set of decisions is discrete and when the set of decisions is continuous. |
| **Learning outcomes:**<br>Students will get understanding of one significant area of operations researches, and will be capable of independent scientific work in this field. The students should be able to recognize the multiple criteria optimization problems in practice, define appropriate mathematical models, and solve them using the aquired knwledge. |
| **Course description (outline):**<br>*Theoretical classes*<br>Introduction and basic terminology. Scalarization techniques: weighted sum method, the epsilon-constraint method, the hybrid method, the elastic constraint method, Benson’s method. Nonscalarizing methods (lexicographic, max-ordering). Multiple criteria linear and non-linear programming. Multiple objective combinatorial optimization.<br>*Practice classes* |
| **References:**<br>[1] Matthias Ehrgott, Multicriteria Optimization, second edition, Springer Berlin Heidelberg New York, 2005.<br>[2] Salvatore Greco, Matthias Ehrgott, Jose Rui Figueira (Eds.), Multiple Criteria Decision Analysis: State of the Art Surveys, second edition, Springer, 2016.<br>[3] Panos M. Pardalos, Antanas Žilinskas, Julius Žilinskasanos. Non-Convex Multi-Objective Optimization, Springer International Publishing, 2017. |
| Active teaching hours: 5 — Theoretical classes: 5 — Practice classes: |
| **Methods of teaching:**<br>Classic lectures including students‘ interaction. Students' knowledge will be tested through homework and seminar papers. The final exam checks the comprehensive understanding of the covered topics. |
| **Grading structure (100 points)**<br>pre-exam activities: 10 marks; seminar work: 30 marks. Oral exam: 60 marks. |

| **Course title:** Metaheuristic Methods | | |
|---|---|---|
| **Teacher:** Tatjana Davidović | | |
| **Course Status:** elective | | |
| **ECTS value:** 12 | | |
| **Requirement:** | | |
| **Course Goals and Objectives:** Introduction to optimization problems and their role in everyday life. Acquiring knowledge about modern optimization methods, especially about approximate (heuristic) methods of discrete (combinatorial) and continuous optimization. Enabling students to recognize, formulate and solve numerous problems in the field of combinatorial optimization using metaheuristic methods. | | |
| **Course Outcome:** Students will acquire the theoretical knowledge necessary to understand the problems related to discrete optimization and the ability to identify problems to which it is necessary to apply heuristic methods. They will be trained to efficiently implement some metaheuristic methods and choose the right method for a specific problem. Implementation will include sequential and parallel computer systems. | | |
| **Course Content**<br>*Theory*<br>Combinatorial and Continuous Optimization Problems; Exact Optimization Methods; Classical Heuristics (constructive and iterative); Metaheuristics (Simulated Annealing, Tabu Search, Variable Neighborhoos Search, Genetic Algorithms, Ant Colony Optimization, Bee Colony Optimization, Particle Swarm Optimization); Hybrid Metaheuristics, Matheuristics. Examples of applications: Travelling Salesman Problem, Scheduling and Routing Problems, Clustering Problem, Location Problems.<br>*Practice* | | |
| **Recommended Literature:**<br>[1] Talbi, El-Ghazali. Metaheuristics: from design to implementation. Vol. 74. John Wiley & Sons, 2009.<br>[2] Gendreau, Michel, and Jean-Yves Potvin, eds. Handbook of metaheuristics. New York: Springer, 2010.<br>[3] Yang, Xin-She. Nature-inspired metaheuristic algorithms. Luniver press, 2010.<br>[4] Maniezzo, Vittorio, Thomas Stützle, and Stefan Voss. Matheuristics: hybridizing metaheuristics and mathematical programming. New York: Springer, 2009. | | |
| Active teaching hours: 5 | Theory: 5 | Practice: |
| **Applicable Teaching Methods**<br>Lectures, homework, seminar papers, consultations. | | |
| **Grading Scheme (max. 100 points)**<br>in-class activity (homework) 10 points, seminar papers 30 points, final exam (oral exam) 60 points | | |

**Course:** Distributed optimization

**Course instructors:** Dusan Jakovetic

**Course type:** elective

**Credit points ECTS:** 12

**Prerequisites: -**

**Course objectives:**

Introducing a wide range of modern optimization methods for large scale, parallel, and distributed optimization; their convergence analysis; and guidelines towards practical implementations.

**Learning outcomes:**

**Outcomes:**

- Ability and experience in applying the taught algorithms on real-world problems;
- Ability to apply the taught algorithms on research problems from a wide variety of application areas;
- Ability to synthesize and analyze efficient distributed algorithms for a given application.

**Course description (outline):**

*Theoretical classes*

Modern first-order methods for large-scale optimization: proximal gradient; accelerated Nesterov gradient; accelerated gradient for non-smooth optimization; Randomized methods: randomized coordinate gradient; stochastic/online gradient; Parallel and distributed methods: primal decomposition; dual decomposition; augmented Lagrangian; ADMM; average consensus methods; distributed gradient; distributed dual averaging; distributed approximate Newton; distributed primal-dual methods; analysis under various network settings (static/time-varying; undirected/directed); convergence and convergence rate analysis under various function settings (smooth/nonsmooth; convex/nonconvex; strongly convex/Lipschitz gradient); connection with training machine learning models; application examples of selected methods on real-world problems.

*Practice classes*

Implementation of the theoretically analysed methods.

**References:**

Main:

Selected papers in the field of distributed optimization

Textbooks (additional)


1. S. Boyd, N. Parikh, E. Chu, B. Peleato, and J. Eckstein, Distributed optimization and statistical learning via the alternating direction method of multipliers, Foundations and Trends in Machine Learning, Vol. 3, No. 1, pp. 1–122, 2011.
2. S. Boyd and L. Vandenberghe, Convex Optimization, Cambridge University Press, 2004.
3. D. Bertsekas, Nonlinear Programming, Athena Scientific, 2004/

D. Bertsekas and J. Tsitsiklis, Parallel and Distributed Computation: Numerical Methods, Prentice-Hall, 1989


| Active teaching hours: 5 | Theoretical classes: 5 | Practice classes: 0 |
|---|---|---|

**Methods of teaching:**

Theoretical lectures and independent work of students during practical hours.

**Grading structure (100 points)**

50 points on pre-exam and 50 points on oral exam

**Course:** Time-varying nonlinear optimization

**Course instructors:** Predrag S. Stanimirović

**Course type:** elective

**Credit points ECTS:** 12

**Prerequisites: -**

**Course objectives:**

Introduce gradient and Zhang dynamical systems for solving time-varying nonlinear optimization problems. Introduce students to the principles of modeling, interpretation and solving real problems by reducing them to the problems of linear and nonlinear optimization.

**Learning outcomes:**

Students trained in the application of nonlinear optimization methods in mathematics, computer science, practice as well as in scientific research and practical applications. Students are expected to recognize problems from science or engineering practice, define appropriate mathematical models as well as solve defined models using learned methods and software packages.

**Course description (outline):**

*Theoretical classes*

- Unconstrained optimization: One-dimensional optimization, non-gradient and gradient optimization methods, Newton method, quasi-Newton methods, convergence rate, conjugate gradient methods.
- Line search and trust-region methods for unconstrained optimization problems.
- Time-varying nonlinear optimization: gradient methods, dynamic system, asymptotic convergence, finite-time and fixed-time convergence in continuous-time optimization, gradient neural network, zeroing neural network.
- Gradient dynamical systems, Zhang dynamical systems, discretization of continuous models, connection with Newton's methods, scalar, vector and matrix models.
- Time-varying matrix inversion and generalized inversion of time-dependent matrices, square root of a matrix, calculation of matrix functions.
- Continuous-time nonlinear optimization with constraints, an approach based on recurrent neural networks.
- Overview of constraint optimization methods (active set methods, sequential quadratic programming, internal point methods, penalty function methods).
- Solving systems of nonlinear equations by methods of nonlinear programming.
- Application of nonlinear optimization in image restoration, robotics, signal processing, solving location problems, solving some problems in economy.

*Practice classes*

Имплементација теоријски обрађених метода

**References:**


1. Y. Zhang, D. Guo, Zhang Functions and Various Models, Springer, 2015.
2. Y. Zhang, L. Xiao, Z. Xiao, M. Mao, Zeroing Dynamics, Gradient Dynamics, and Newton Iterations, Taylor & Francis Group, 2016.
3. Y. Zhang, C. Yi, Zhang Neural Networks and Neural-dynamic Method, Nova Science Publishers, 2011.
4. Y. Wei, P.S. Stanimirović, M. Petković, Numerical and symbolic computations of generalized inverses, World Scientific Publishing Co. Pte. Ltd., Hackensack, NJ, 2018, September 2018, DOI 10.1142/10950


| | | |
|---|---|---|
| Active teaching hours: 5 | Theoretical classes: 5 | Practice classes: |

**Methods of teaching:** предавања и самостални рад студената на практичним часовима

**Grading structure (100 points):** 50 предиспитне обавезе, 50 испит

**Course title:** Graph Theory

**Teacher:** Bojana D. Borovićanin

**Course Status:** elective

**ECTS value:** 12

**Requirement:**

**Course Goals and Objectives:** Introduction to the concepts and theorems of graph theory, as well as some possibilities of its application. Enabling students to formulate and solve numerous problems in this field using graph theory techniques and methods.

**Course Outcome:** The student has acquired the theoretical knowledge necessary to understand the problems in graph theory, including possible applications in mathematics, computer science, electrical engineering, natural sciences, and other fields. The student has mastered the skills and methods of research in this area.

**Course Content**

*Theory*

Basic concepts of graph theory. Graph invariants. Graph operations. Planar graphs and polyhedron graphs. Euler's theorem, Kuratowski–Pontryagin theorem. Graph colouring. Chromatic number of graphs. Euler and Hamilton paths and contours. Independent sets, covers, and graph cliques. Internal and external stability of graphs with application in code theory. Menger's theorem and transport networks. Matrices in graph theory. Linear algebra and graphs. Groups and graphs. Extreme graphs. General method of defining different types of graph spectra. Coefficients of different characteristic polynomials of a graph. Graph operations and resulting spectra. Relationships between spectral and structural properties of digraphs and graphs. Graph eigenvectors. Characterization of graphs by spectra. Spectral techniques in graph theory. Application in computing, chemistry, and physics.

*Practice*

**Recommended Sources**


1. L. Beineke, R. Wilson, P. Cameron, *Topics in Algebraic Graph Theory*, Cambridge University Press, Cambridge, 2004.
2. B. Bollobas, *Modern Graph Theory*, Series: Graduate Texts in Mathematics, Vol. 184, Springer, New York, 1998.
3. D. Cvetković, *Teorija grafova i njene primene*, Naučna knjiga, Beograd, 1981.
4. D. Cvetković, M. Doob, H. Sachs, *Spectra of Graphs*, 3rd edition, Johann Ambrosius Barth Verlag, Heidelberg–Leipzig, 1995.
5. R. Diestel, *Graph Theory*, Series: Graduate Texts in Mathematics, Vol. 173, Springer, Berlin, Heidelberg, 2017.
6. V. Petrović, *Teorija grafova*, Univerzitet u Novom Sadu, 1998.
7. D. West, *Introduction to Graph Theory*, Second Edition, Prentice Hall, 2001.


| Active teaching hours: 5 | Theory: 5 | Practise: |
|---|---|---|

**Applicable Teaching Methods**

Lectures, homework, seminar papers, consultations

**Grading Scheme (max. 100 points)**

in-class activity (homework) 10 points, seminar papers 30 points, final exam (oral exam) 60 points

**Course:** Stochastic optimization

**Course instructors:** Nataša Krklec Jerinkić

**Course type:** elective

**Credit points ECTS:** 12

**Prerequisites: -**

**Course objectives:**
Introducing basic concepts of stochastic optimization, analysis of basic algorithms and their modifications.

**Learning outcomes:**

- Acquiring basic concepts of stochastic optimization;
- Ability of applying the taught algorithms on real-world problems;
- Ability to construct and analyze stochastic optimization methods.

**Course description (outline):**

*Theoretical classes*

Introducing stochastic optimization problems, noisy objective functions, stochastic constraints, probabilistic (chance) constraints, functions in the form of mathematical expectation. Concept of stochastic convergence. Approximations of functions in stochastic environment, Stochastic Approximation (SA) methods and Sample Average Approximation (SAA) methods. Convergence analysis of SA methods. Sampling methods for noisy functions approximations. Approximations of the derivatives, finite differences, simultaneous perturbations. Convergence analysis and statistical properties of SAA methods. Validation analysis. Concept of adaptive sample size methods.

*Practice classes*

Implementation of the theoretically analysed methods.

**References:**

Main:


1. Shapiro, A., Dentcheva, D. and Ruszczynski, A., 2021. *Lectures on stochastic programming: modeling and theory*. Society for Industrial and Applied Mathematics.
2. Spall, J.C., 2005. *Introduction to stochastic search and optimization: estimation, simulation, and control* (Vol. 65). John Wiley & Sons.


Textbooks (additional)

Selected papers in the field of stochastic optimization

| Active teaching hours: 5 | Theoretical classes: 5 | Practice classes: |
|---|---|---|

**Methods of teaching:**
Theoretical lectures and independent work of students during practical hours.

**Grading structure (100 points)**
50 points on pre-exam and 50 points on oral exam

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|---|
| **Course:** Introduction to Machine Learning |
| **Course instructors:** Lazar Velimirović |
| **Course type:** elective |
| **Credit points ECTS:** 12 |
| **Prerequisites:** |
| **Course objectives:**<br>Broader introduction to the principles, algorithms and basics of machine learning from the angle of modeling and prediction. The main topics include linear and nonlinear models for supervised and unsupervised learning. |
| **Learning outcomes:**<br>The student will be familiar with the broader basics, principles and algorithms of machine learning. The student will be able to recognize, model and implement different machine learning algorithms, as well as to evaluate the performance of different models. |
| **Course description (outline):**<br>• Linear Regression, parameter estimation, predictions and models, non-linear regression<br>• Logistic Regression, maximum likelihood, classification<br>• Penalty regression, LASSO, Ridge, regularization<br>• Generative models, Naïve Bayes, conditional probabilities<br>• Divide and conquer, decision trees, random forests, nearest neighbors<br>• Representational/factor models, PCA,SVD, matrix factorization<br>• Perceptron, support vector machine, constrained optimization, kernels<br>• Neural networks, Convolution neural networks |
| **References:**<br>8. Mohri, M., Rostamizadeh, A., & Talwalkar, A. (2018). Foundations of machine learning. MIT press.<br>9. Shalev-Shwartz, S., & Ben-David, S. (2014). Understanding machine learning: From theory to algorithms. Cambridge university press. |
| Active teaching hours: 5 — Theoretical classes: 5 — Practice classes: |
| **Methods of teaching:**<br>Classical teaching methods with video projectors and interaction with students. Students' knowledge is tested through homework and defense of seminar papers. The final oral exam checks the comprehensive understanding of the presented material. |
| **Grading structure (100 points)**<br>Pre-exam obligations:<br>• activity during classes 10 points,<br>• seminar paper or oral seminar 20 points,<br>Oral exam 70 points |

**Course title:** Artificial Neural Networks

**Teacher or teachers:** Branimir T. Todorovic

**Course status:** elective

**ECTS number:** 12

**Condition: no**

**The aim of the course**

Acquiring knowledge in the field of artificial neural networks and their application in processing in the analysis and processing of images, text, time series, video and audio signals.

**Outcome of the case**

At the end of the course the student should be able to choose an algorithm for adapting the architecture and parameters of artificial neural networks, implement them using some of the available software environments in Python (NumPy, CuPy, PyTorch) and apply in solving problems in intelligent image processing, text , time series, video and audio signals**.**

**Course content**

Mathematical models of neurons, layers with direct signal propagation, matrix form of backward error propagation, normalization and regularization of layers, recurrent neural networks, papacy of error back time, Kalman filter as a neural network learning algorithm, convolutional neural networks, backward propagation of error through convolutional layers, focus layers, autoencoders, variational autoencoders, generative adversary networks, development and coding of software environment in Python for implementation of artificial neural networks using NumPy and CuPy libraries, calculation graphs, algorithmic differentiation, implementation of direct propagation and error propagation backward through linear normalizations, convolutional layer, deconvolutional layer, recurrent layer. Application in word processing, time series prediction, identification and control of dynamic systems, image processing, video and audio signals.

**Recomended literature**

1. Deep Learning, I. Goodfellow, Y. Bengio, A. Courville, MIT Press, 2016
2. Neural Networks and Deep Learning, 2018, Charu C. Aggarwal, Springer, ISBN-13: 978-3319944623, ISBN-10: 3319944622
3. B. Todorović, S. Todorović-Zarkula, M. Stanković, Rekurentne neuronske mreže: estimacija parametara, stanja i strukture, Univerzitet u Nišu, Prirodno-matematički fakultet, 2012.
4. Samuel Burns, Python Deep learning: Develop your first Neural Network in Python Using Tensor Flow, Keras, and PyTorch, Independently Published, 2019, ISBN-13: 978-1092562225, ISBN-10: 1092562222

| **Number of hours of active teaching** | **Theoretical teaching:** 5 | **Practical teaching:** |
|---|---|---|

**Teaching methods**

The lectures use classical teaching methods with the use of video projectors and interaction with students. Students' knowledge is tested through homework and projects. The final oral exam checks the comprehensive understanding of the presented material.

**Assessment of knowledge (maximum number of points 100)** colloquia - 30, seminars - 20, oral exam - 50

| |
|---|
| **Course:** Linear partial differential equations |
| **Course instructors:** Marko Nedeljkov |
| **Course type:** elective |
| **Credit points ECTS:** 12 |
| **Prerequisites:** none |
| **Course objectives:**<br>Invite students into the theory of linear PDEs |
| **Learning outcomes:**<br>Understanding of the basic principles of linear PDEs analysis |
| **Course description (outline):**<br>*Theoretical classes*<br>*Characteristics. Holmgrens Theorem , harmonic analysis with applications. Distributions, Sobolev spaces. Wave, heat, Laplace, Schroedinger equations. Energy inegral, maximum principles* |
| **References:**<br>1. *J. Rauch. Partial Differential Equations, Springer 1992.*<br>2. L.C. Evans, Partial Differential Equations, II ed, AMS 2012 |

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|---|---|---|
| Active teaching hours : 5 | Theoretical classes: 5 | Practice classes: |

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| **Methods of teaching:**<br>Lectures and independent work of students |
| **Grading structure (100 points)**<br>50 Colloquia, 50 Exam |

**Course:** Numerical Analysis of Partial Differential Equations

**Course instructors:** Marko Nedeljkov

**Course type:** elective

**Credit points ECTS:** 12

**Prerequisites:** Linear PDEs

**Course objectives:**
Learning basic methods of solving numericaly PDEs.

**Learning outcomes:**
Students will have knowledge of fiinite difference methods and other solution methods and a way to applly that knowledge.

**Course description (outline):**
*Theoretical classes*
*Ellipti boundary value problems: existene and uniqueness of weak solutions. Finite di#erene approximation of ellipti boundary value problems. Finite element methods for elliptic boundary value problems. Finite di#erene approximation of evolutionary problems.*

**References:**


1. E. Suli, An Introduction to a Numerical Analysis of PDEs, Oxford 2005.
2. *J.W. Thomas, Numerical Partial Differetial Equations, Finite Difference Methods, Springer, 1995.*


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|---|---|---|
| Active teaching hours : 5 | Theoretical classes: 5 | Practice classes: 0 |

**Methods of teaching:**
Lectures and independent work of students

**Grading structure (100 points)**
50 Colloquia, 50 Exam

| **Course:** Hyperbolic partial differential equations | | |
|---|---|---|
| **Course instructors:** Marko Nedeljkov | | |
| **Course type:** elective | | |
| **Credit points ECTS:** 12 | | |
| **Prerequisites:** Linear partial differential equations | | |
| **Course objectives:**<br>Dealing with different types of hyperbolic equations | | |
| **Learning outcomes:**<br>Students will have knowledge that enables research work with hyperbolic equations and systems, as well as the applications in other scientific fields. | | |
| **Course description (outline):**<br>*Theoretical classes*<br>*Part A. Linear equations and systems: Notions of hyperbolicity, well-posedness of the Cauchy problem, distributional solutions, methods from semigroups of operators; examples from physics and geophysics. Part B. Nonlinear equations and conservation laws: Characteristics, weak solutions, entropy con- ditions, methods from nonlinear semigroups; examples from physics and geophysics.* | | |
| **References:**<br>1. F. Treves, Basic linear partial differential equations, Academic Press 1975.<br>2. L. Hörmander, The analysis of linear partial differential operators, volume II, Springer 1983.<br>3. A. Pazy, Semigroups of linear operators and applications to partial differential equations, Springer 1983.<br>4. S. Benzoni-Gavage and D. Serre, Multi-Dimensional Hyperbolic Partial Differential Equations: First-Order Systems and Applications, Oxford Uni Press 2007.<br>5. L. C. Evans, Partial differential equations, Amer. Math. Soc., 2nd edition 2010.<br>6. V. Barbu, Nonlinear differential equations of monotone types in Banach spaces, Springer 2010.<br>7. C. M. Dafermos, Hyperbolic conservation laws in continuum physics, Springer, 4th edition 2016. | | |
| Active teaching hours: 5 | Theoretical classes: 5 | Practice classes: |
| **Methods of teaching:**<br>Lectures and independent work of students | | |
| **Grading structure (100 points)**<br>50 Colloquia, 50 Exam | | |

| | | |
|---|---|---|
| **Course:** Finite Element Method | | |
| **Course instructors:** Nataša Krejić | | |
| **Course type:** elective | | |
| **Credit points ECTS:** 12 | | |
| **Prerequisites:** Numerical analysis of PDEs | | |
| **Course objectives:**<br>Learning Finite Element Method (FEM) | | |
| **Learning outcomes:**<br>Students will have knowledge of FEM that enables their research work, as well as the applications in other scientific fields. | | |
| **Course description (outline):**<br>*Theoretical classes*<br>*Function spaces. Approximation of elliptic problems. Piecewise polynomial approximation. A posteriori error analysis by duality. Evolution problems.* | | |
| **References:**<br>1. *E. Suli, Lecture Notes on Finite Element Methods for Partial Differential Equations, Oxford 2020.*<br>2. *A. Ern and J.-L. Guermond, Theory and practice of finite elements, vol. 159 of Applied Mathematical Sciences, Springer-Verlag, New York, 2004* | | |
| Active teaching hours: 5 : | Theoretical classes: 5 | Practice classes: |
| **Methods of teaching:**<br>Lectures and independent work of students | | |
| **Grading structure (100 points)**<br>50 Colloquia, 50 Exam | | |

| | | |
|---|---|---|
| **Course:** Mathematical Methods in Continuum Mechanics | | |
| **Course instructors:** Srboljub Simić, Milana Čolić | | |
| **Course type:** | | |
| **Credit points ECTS:** | | |
| **Prerequisites:** | | |
| **Course objectives:**<br>Introduction to basic principles of mechanics and thermodynamics of continua, mathematical methods useed in this field and their application in modelling and analysis of diverse processes in continuous media. | | |
| **Learning outcomes:**<br>It is expected that student develop the skills of application of basic principles of continuum mechanics in modelling the processes in continuous media, and the use of appropriate mathematical methods in their analysis, especially mathematical analysis and partial differential equations. | | |
| **Course description (outline):**<br>*Theory*<br>Vector and tensor algebra and analysis. Kinematics of continua. Basic principles and basic equations of continuum mechanics and thermodynamics: conservation laws and balance laws, entropy inequality; integral and local form of basic equations/weak and strong form of the equations. Thermodynamic analysis of shock waves. Constitutive relations: mathematical description material response, mechanical and thermodynamic restrictions.<br>*Applications*<br>Mathematical analysis of particular processes in continuous media: heat conduction in rigid bodies; compressible and incompressible fluid flow; linear and nonlinear elasticity; thermoelasticity; linear viscoelasticity; fluid flow through porous media, diffusion. | | |
| **References:**<br>1. M.E. Gurtin, E. Fried, L. Anand: *The Mechanics and Thermodynamics of Continua*, Cambridge University Press, Cambridge 2010.<br>2. R. Temam, A. Miranville: *Mathematical Modeling in Continuum Mechanics,* Cambridge University Press, Cambridge 2005.<br>3. C.M. Dafermos: *Hyperbolic Conservation Laws in Continuum Physics*, 4th Edition, Springer-Verlag, Berlin 2016.<br>4. A.C. Fowler: *Mathematical Models in the Applied Sciences*, Cambridge University Press, Cambridge 1997. | | |
| Activet eaching hours: | Theoretical classes: 5 | Practice classes: 5 |
| **Methods of teaching:**<br>Teaching is organized in a combined way, in-class lectures and individual student’s research. In-class lectures are devoted to the analysis of theoretical aspects of mathematical models and methods applied in continuum mechanics. Individual student’s research is focused on application of theoretical results to the analysis of diverse specific problems. The choice of the problems is based upon student’s research preferences. | | |
| **Grading structure (100 points)** | | |
| Предиспитне обавезе/семинар-и: 50 поена<br>Усмени испит: 50 поена | | |

**Course:** Mathematical aspects of quantum physics

**Course instructors:** Marko Nedeljkov

**Course type:** elective

**Credit points ECTS:** 12

**Prerequisites:** Functional analysis

**Course objectives:**
Invite students into the mathematical theory of quantum physics.

**Learning outcomes:**
Understanding of the basic principles of quantum physics.

**Course description (outline):**
*Theoretical classes*
*Unitary groups of operators on Hilbert spaces, axioms of quantum mechanics, Schrödinger equation, applications of spectral properties of self-adjoint operators, bound states and scattering states, angular momentum, the simplest atomic systems; Dirac equation.*

**References:**

1. M. Reed and B. Simon, Methods of modern mathematical physics, 4 volumes, Ac.Press 1975-80.
2. B. C. Hall, Quantum theory for Mathematicians, Springer 2013.
3. M. Schechter, Operator methods in quantum mechanics, Elsevier 1981.
4. B. Thaller, The Dirac equation, Springer 1992.
5. G. B. Folland, Quantum field theorya tourist guide for mathematicians, Amer. Math. Soc. 2008.
6. W. Thirring, Quantum mathematical physicsatoms, molecules and large systems, Springer, 2nd ed. 2002.


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|---|---|---|
| Active teaching hours: 5 | Theoretical classes: 5 | Practice classes: |

**Methods of teaching:**
Lectures and independent work of students

**Grading structure (100 points)**
50 Colloquia, 50 Exam

**Course:** Symmetry group analysis of differential equations

**Teacher(s):** Sanja Konjik

**Course status:** elective

**ECTS:** 12

**Prerequisites:** None

**Goal:** Acquiring knowledge and skills from selected topics of symmetry group analysis of differential equations

**Outcomes:** Student is able to independently follow the achievements in the field of symmetry group analysis and to apply the acquired knowledge and skills to specific problems

**Contents**

*Theoretical lectures*

Transformation of manifold, Lie group, group of transformations, Lie group of transformations, local vector fields, local diffeomorphism, orbits, distributions on manifolds, rank, involutive sets and distributions, integral manifolds of distribution, integrable distributions, infinitesimal automorphism of distribution, initial submanifold, Frobenius theorem, local Lie transformation group, orbit of the local transformation group, infinitesimal generator, connected, semi-regular and regular local transformation groups, local and global G-invariance, infinitesimal criterion, Hadamard‘s lemma, functionally dependence, Buckingham Pi-theorem, symmetry group, prolongations, maximal rank, total derivative, prolongation formula, calculation of symmetry groups, local solvability, nondegeneracy conditions, integration of ODEs, differential invariants, variational problems, variational derivative, Euler‘s operator, Euler-Lagrange equations, variational symmetry, total divergence, infinitesimal criterion for variational symmetries, conservation laws, Noether‘s theorem, infinitesimal divergence symmetry, characteristics

**Recommended bibliography**


1. Kunzinger, M., Lie Transformation Groups - An Introduction to Symmetry Group Analysis of Differential Equations, Lecture notes, University of Vienna, 2015.
2. Olver, P.J., Applications of Lie Groups to Differential Equations, 2nd edition, Springer, New York, 2000.
3. Olver, P.J., Equivalence, Invariants, and Symettry, Cambridge University Press, Cambridge, 2009.
4. Warner, F.W., Foundations of Differentiable Manifolds and Lie Groups, Springer-Verlag, New York, 1983.
5. Hall, B., Lie Groups, Lie Algebras, and Representations - An Elementary Introduction, Springer, Switzerland, 2015.


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| Active teaching hours: 5 | Theoretical: 5 | Practical: |

**Methods of teaching:**

Theoretical lectures and individual work of students during practical hours

Knowledge estimation: **(max 100 points)**

Mini-project 20 points, oral exam 80 points